Additive Cubic-Density Constraints on Minimal Counterexamples to the Erdős–Gyárfás Conjecture

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Status. This is an AI-assisted research draft seeking independent human mathematical and computational review. The candidate results below were developed with extensive use of AI systems and have undergone multiple AI-assisted adversarial checks, independently implemented verification routes, and reruns described in the accompanying code package. The human author has not independently reconstructed all mathematical arguments or locally reproduced all computations. No independent human expert verification has yet been completed, and no priority claim is made. The paper does *not* resolve the Erdős–Gyárfás conjecture.

Abstract

The Erdős–Gyárfás conjecture asks whether every finite simple graph of minimum degree at least three contains a cycle whose length is a power of two. Suppose that a counterexample G exists and choose one first with minimum order and, subject to that, minimum size. Write

$$a = |V_3(G)|, \qquad b = |V_{\geq 4}(G)|.$$

Building on the minimal-counterexample structure developed by Markström and Carr, we obtain two computer-assisted additive constraints on the degree distribution. First,

$$\boxed{a \geq 2b + 8}$$

for every such minimal counterexample. Equivalently,

$$|V_3(G)| \geq \left\lceil \frac{2|V(G)| + 8}{3} \right\rceil.$$

Second, in the range $b \geq 14$ we prove the stronger bound

$$\boxed{a \geq 2b + 10}.$$

The proof suppresses certain cubic vertices to a simple 2-degenerate graph Q on the high-degree vertices and uses the exact identity

$$a = 2b + d + E + z, \qquad d = 2b - e(Q), \qquad d \equiv z \pmod 2.$$

For large b, fixed-defect extremal bounds on Q suffice. The remaining small orders are reduced to finite families. Their lift obstructions are verified using independent forbidden-cycle implementations, exact component enumerations, and small integer charging certificates. For the $+10$ theorem, the final path-only obstruction consists of 1943 static endpoint-budget certificates $\lambda : E(Q) \to \{0, 1, 2\}$. Full source code, finite case data, certificates, hashes, and reproduction scripts are separated from the manuscript in the accompanying reproducibility package.

1 Introduction

The Erdős–Gyárfás conjecture asserts that every finite simple graph of minimum degree at least 3 contains a simple cycle whose length is a power of two [1]. The conjecture remains open. A natural way to attack it is to assume a counterexample exists and ask how rigid a minimal one must be.

Let G be a counterexample chosen with the minimum possible number of vertices and, subject to that, the minimum possible number of edges. Markström established early computational and structural restrictions on such graphs [2]. Carr recently proved, among other things, that vertices of degree at least four form an independent set, that every vertex has a cubic neighbour, and that at least $4/7$ of the vertices of a minimal counterexample have degree three [3]. Our aim is to strengthen the cubic-density conclusion by exploiting a suppression graph together with finite exact checks.

Put

$$A = V_3(G), \qquad B = V_{\geq 4}(G), \qquad a = |A|, \qquad b = |B|.$$

Our main candidate results are the following.

Theorem 1 (Universal additive +8)**.** *Every minimal counterexample satisfies*

$$\boxed{a \geq 2b + 8.}$$

Equivalently, with $n = |V(G)| = a + b$*,*

$$\boxed{a \geq \left\lceil \frac{2n+8}{3} \right\rceil.}$$

Theorem 2 (Additive +10 for $b \geq 14$)**.** *Every minimal counterexample with* $b \geq 14$ *satisfies*

$$\boxed{a \geq 2b + 10.}$$

Equivalently,

$$a \geq \left\lceil \frac{2n+10}{3} \right\rceil \qquad \textit{whenever } b \geq 14.$$

These statements are conditional structural restrictions: they say what a hypothetical minimal counterexample must look like. Neither theorem proves that no counterexample exists.

The proof has two layers. The common structural layer derives an exact suppression identity and parity relation. The computational layer then proves fixed-defect extremal bounds for the suppression graph and eliminates a finite number of exceptional lift configurations. The universal theorem requires separate finite treatments at $b \leq 10$; the +10 theorem has a much more concentrated final obstruction at $b \in \{14, 15\}$.

2 Minimal-counterexample structure

We use the following consequences of Carr's minimal-counterexample argument [3].

Lemma 3 (Carr's structural consequences)**.** *Let* G *be a minimal counterexample. Then:*

(i) every proper subgraph $H \subsetneq G$ *has minimum degree at most* 2*;*

(ii) every vertex of G *has a neighbour of degree exactly* 3*;*

(iii) $B = V_{\geq 4}(G)$ *is an independent set.*

Let $F = G[A]$. By Lemma 3(ii), every vertex of A has positive degree in F. Write

$$x = |\{u \in A : d_F(u) = 1\}|, \qquad y = |\{u \in A : d_F(u) = 2\}|, \qquad z = |\{u \in A : d_F(u) = 3\}|.$$

Define the excess degree on B by

$$E = \sum_{v \in B} (d_G(v) - 4) \geq 0.$$

Because B is independent, all degree from B goes to A. Counting A–B incidences from both sides gives

$$2x + y = 4b + E. \tag{1}$$

Since $a = x + y + z$,

$$a = 4b + E + z - x. \tag{2}$$

3 The suppression graph and the defect identity

For every $u \in A$ with $d_F(u) = 1$, the other two neighbours of u lie in B. Suppress u by replacing the length-two path between those two vertices of B by a single edge. Performing this for every such u produces a graph Q on B.

Lemma 4 (Suppression identity)**.** *The graph Q is simple, $e(Q) = x$, and*

$$\boxed{a = 4b + E + z - e(Q).} \tag{3}$$

Moreover

$$\boxed{e(Q) \equiv z \pmod 2.} \tag{4}$$

Proof. A loop cannot arise because the two B-neighbours of a suppressed vertex are distinct. If two suppressed vertices produced the same edge pq, they would form a 4-cycle with p and q in G, impossible. Hence Q is simple and $e(Q) = x$. Equation (3) follows from (2). The handshake lemma in F gives $x + 2y + 3z \equiv 0 \pmod 2$, so $x \equiv z \pmod 2$. □

Lemma 5 (Structure of Q)**.** *The graph Q is 2-degenerate. A simple cycle of length ℓ in Q lifts to a simple cycle of length 2ℓ in G. In particular, Q has no cycle whose length is a power of two.*

Proof. Distinct edges of a simple cycle in Q correspond to distinct suppressed vertices, so expanding each edge doubles the length and gives a simple cycle in G. If some subgraph of Q had minimum degree at least 3, it would be a smaller graph of minimum degree at least 3 with no power-of-two cycle, contradicting the minimality of G. □

It is convenient to define the suppression defect

$$d := 2b - e(Q).$$

Combining Lemma 4 with this definition yields the form used throughout the paper:

$$\boxed{a = 2b + d + E + z}, \qquad \boxed{d \equiv z \pmod 2}. \tag{5}$$

4 Fixed-defect extremal bounds for the suppression class

Let $\mathcal{H}$ be the hereditary class of simple, 2-degenerate, C_4, C_8-free graphs. Every suppression graph Q belongs to $\mathcal{H}$.

Lemma 6 (Deletion bootstrap)**.** *Fix integers N and c. If no graph in $\mathcal{H}$ on N vertices has $2N-c+1$ edges, then for every $H \in \mathcal{H}$ with $|V(H)| \geq N$,*

$$e(H) \leq 2|V(H)| - c.$$

Proof. If a counterexample existed, delete edges until equality $e(H) = 2|V(H)| - c + 1$ holds. Since H is 2-degenerate, delete a vertex of degree at most two. The remaining graph has at least $2(|V(H)| - 1) - c + 1$ edges, remains in $\mathcal{H}$, and may again be edge-deleted to equality. Iterating reaches order N, contradiction. □

The finite reconstruction used in the proof gives the following extremal values for the small orders needed by Theorem 1:

b	4	5	6	7	8	9	10	11
$q(b) := \max\{e(H) : H \in \mathcal{H},\ \|V(H)\| = b\}$	4	6	7	9	11	12	14	15

In particular, the complete 11-vertex reconstruction contains 245 isomorphism classes with 15 edges and none with 16 edges. Lemma 6 gives

$$\boxed{e(Q) \leq 2b - 7 \qquad (b \geq 11).} \tag{6}$$

For Theorem 2, two stronger fixed-defect facts are used:

$$\boxed{e(Q) \leq 2b - 8 \qquad (b \geq 14),} \tag{7}$$

$$\boxed{e(Q) \leq 2b - 9 \qquad (b \geq 16).} \tag{8}$$

The finite bases are: no 14-vertex member of $\mathcal{H}$ has 21 edges, and no 16-vertex member of $\mathcal{H}$ has 24 edges. For later finite reductions, the complete families with $e = 2b - 8$ contain 852 isomorphism classes at $b = 14$ and 168 classes at $b = 15$.

All finite extremal data in this section are regenerated from the packaged base representatives by independent closure code. They are mathematical inputs to the computer-assisted part of the proof, not heuristic search statistics.

5 Proof of the universal additive +8 theorem

We now prove Theorem 1, splitting by b.

5.1 The range $b \geq 11$

By (6), $d \geq 7$. Since $d \equiv z \pmod 2$, either d is even and therefore $d \geq 8$, or d is odd and $z \geq 1$. Hence in every case

$$d + z \geq 8.$$

Equation (5) then gives

$$a = 2b + d + E + z \geq 2b + 8.$$

Thus only $b \leq 10$ requires further work.

5.2 The orders $b = 9, 10$

The exact values $q(9) = 12$ and $q(10) = 14$ imply $d \geq 6$. Suppose $a < 2b + 8$. Then

$$d + E + z \leq 7, \qquad d \equiv z \pmod 2,$$

so the only possibility is

$$d = 6, \qquad z = 0, \qquad E \in \{0, 1\}.$$

The complete extremal suppression families consist of 33 non-isomorphic $(9, 12)$ cores and 14 non-isomorphic $(10, 14)$ cores. Degree feasibility gives exactly

$$22 + 206 + 4 + 48 = 280$$

labelled cases in the sectors $(b, E) = (9, 0), (9, 1), (10, 0), (10, 1)$.

Because $z = 0$, every component of F is a path or a cycle. For a fixed labelled case, let $t_v = 4 + \varepsilon_v - d_Q(v)$ be the capacity of a label $v \in B$, where $\sum_v \varepsilon_v = E$. Define the bad-pair graph D_Q on positive-capacity labels by joining p, q if Q has neither a simple p–q path of length 2 nor one of length 6. If labels at F-distance two are equal, a C_4 appears; if they are distinct but joined by a length-2 or length-6 path in Q, the suppression expansion produces a C_8 or C_{16}. Hence the step-two label sequence of every F-cycle gives a closed walk in D_Q.

Two independently implemented finite checks show that every D_Q in the 280 cases is a forest. The only potentially surviving internal cycle has length 12, but the maximum total capacity on vertices incident with D_Q is respectively $7, 9, 0, 0$ in the four sectors, below the required 12. Thus every case is path-only.

For paths, let m_{ef} be the maximum number of internal y-vertices in a locally admissible single path between endpoint tokens e and f, where the local model imposes only necessary conditions: per-label capacity bounds and the absence of C_4, C_8, C_{16} in the full suppression skeleton plus that path. Every true path therefore appears in the relaxed local family. The endpoint pairs in a true path decomposition form a matching, and an exact maximum-weight matching computation gives sector maxima

$$10, \ 11, \ 2, \ 4$$

against required values

$$12, \ 13, \ 12, \ 13.$$

Two local-cycle implementations agree. In addition, a separate direct-cycle verifier checks static integer endpoint budgets with maximum sums $11, 12, 2, 4$, again strictly below the required y. Thus all 280 cases are impossible.

5.3 The order $b = 8$

The exact bound $q(8) = 11$ gives $d \geq 5$. Under the contradiction assumption $a < 2b + 8$, (5) and parity reduce to two branches:

$$(d, z) = (5, 1) \quad \text{or} \quad (d, z) = (6, 0), \qquad E \in \{0, 1\}.$$

There are exactly two $(8, 11)$ suppression cores in the first branch and 26 $(8, 10)$ cores in the second.

For $d = 5, z = 1$, an independent graph check verifies that every pair of distinct B-vertices is joined in Q by a simple path of length 2 or 6. Thus no two y-vertices can be at F-distance two. The unique component containing the degree-three vertex of F is necessarily a tripod or a lollipop; the pair-cover property rules out cycles and lollipops. Every ordinary path contains at

most two y-vertices. The tripod also contains at most two: two nonempty arms would place their first y-vertices at F-distance two, while three y-vertices on one arm would place the first and third at distance two. Since $x = 11$, the tripod uses three endpoints and the other eight endpoints form four ordinary paths. Hence at most ten y-vertices occur. This immediately contradicts $y = 11$ when $E = 1$.

When $E = 0$, equality $y = 10$ forces the tripod and all four paths to contain exactly two y-vertices. The final obstruction depends only on the one-subdivision endpoint skeleton $S(Q)$. For one of the two $(8, 11)$ cores every possible pair of short tripod endpoints already has an $S(Q)$-path of length 2, 6, or 14, closing a C_4, C_8, or C_{16}. For the other core the only short-endpoint pair avoiding these lengths is one exceptional token pair; for every choice of the third endpoint, an $S(Q)$-path of length 4 or 12 to a short endpoint, together with the corresponding length-4 route through the tripod, closes a C_8 or C_{16}. A NetworkX implementation verifies these skeleton facts, while an independently written C++ checker directly enumerates all capacity-feasible two-y tripod labelings and finds none.

For $d = 6, z = 0$, degree feasibility gives 184 labelled cases: 20 with $E = 0$ and 164 with $E = 1$. Cycle components are eliminated by two closed-walk/capacity implementations. The remaining path-only cases admit static label-plus-endpoint charging certificates: for each case there are weights

$$\mu_v \in \{0, 1, 2\} \quad (v \in B), \qquad \lambda_e \in \{0, 1, 2\} \quad (e \in E(Q))$$

such that every locally admissible single path P satisfies

$$\sum_v \mu_v c_v(P) \le \lambda_e + \lambda_f,$$

while

$$\sum_v \mu_v t_v > \sum_e \lambda_e.$$

Summing over the true path decomposition gives a contradiction. A direct simple-cycle verifier recomputes all admissible paths from the raw 184 cases and checks every certificate. A separate exact-quota computation provides a supplementary cross-check.

5.4 The orders $b = 4, 5, 6, 7$

The complete graph atlas gives

$$q(4) = 4, \quad q(5) = 6, \quad q(6) = 7, \quad q(7) = 9.$$

Assume $a < 2b + 8$. Then $d + E + z \le 7$. For $b = 6, 7$ the only possible defect branches are

$$(d, z) = (5, 1) \quad \text{or} \quad (d, z) = (6, 0), \qquad E \in \{0, 1\}.$$

For $b = 4, 5$ there are also the branches

$$(d, z) = (4, 0), \quad E \in \{0, 1, 2, 3\},$$

and

$$(d, z) = (4, 2), \quad E \in \{0, 1\}.$$

For the $d = 5$ and $d = 6$ branches, the complete degree-feasible labelled case counts are

b	$(d,z)=(5,1)$	$(d,z)=(6,0)$
4	15	10
5	24	30
6	22	71
7	32	114

When $z=0$, every component of F is a path or a cycle. When $z=1$, the component containing the unique degree-three vertex is exactly a tripod or a lollipop, while every other component is a path or cycle. The global finite search enumerates actual component label sequences, uses every endpoint exactly once, enforces exact label quotas, assembles all selected components in a common graph, and rejects every C_4, C_8, C_{16}, including cycles involving more than one component. Two independent cycle oracles – alternate paths of lengths $3, 7, 15$ and direct simple-cycle enumeration – return UNSAT for every listed case.

For $d=4, z=0$, the unique extremal suppression core at $b=4,5$ has diameter two. Thus two y-vertices cannot be at F-distance two: equal labels produce C_4 and distinct labels close a C_8. There are $b-2$ path components, each with at most two y-vertices, giving

$$y \le 2(b-2) \le 6,$$

whereas $y = 2d + E = 8 + E \ge 8$, contradiction.

For $d=4, z=2$, the same distance-two obstruction excludes pure y-cycles. Suppressing the y-vertices leaves a topological multigraph with $x = 2b-4$ degree-one vertices and two degree-three vertices. Complete half-edge pairing and subdivision enumeration, including topological loops, finds no valid expansion with $y=8$ or 9. The enumeration covers 945 half-edge pairings at $b=4$ and 10,395 at $b=5$.

Thus no violation exists for $4 \le b \le 7$.

5.5 The orders $b \le 3$

The established computational lower bound states that any counterexample has at least 17 vertices [2, 4]. Hence, if $b \le 3$,

$$a = n - b \ge 17 - b \ge 2b + 8.$$

This completes the proof of Theorem 1, subject to the finite certifications described above.

6 The additive $+10$ strengthening for $b \ge 14$

We next prove Theorem 2. The fixed-defect bounds (7)–(8) and parity sharpen the universal estimate. From (7), since the cap $2b-8$ is even,

$$a \ge 2b + E + 8 + 2\left\lceil \frac{z}{2} \right\rceil \qquad (b \ge 14). \tag{9}$$

From (8), whose edge cap is odd,

$$a \ge 2b + E + 10 + 2\left\lfloor \frac{z}{2} \right\rfloor \qquad (b \ge 16). \tag{10}$$

Suppose $b \ge 14$ but $a < 2b + 10$. Equation (9) forces

$$z = 0, \qquad E \in \{0, 1\},$$

and (10) excludes $b \geq 16$. Therefore

$$\boxed{b \in \{14, 15\}, \qquad z = 0, \qquad E \in \{0, 1\}.} \tag{11}$$

In every such case $e(Q) = 2b - 8$ and

$$y = 16 + E.$$

Filtering the 852 cores at $b = 14$ and 168 cores at $b = 15$ by nonnegative degree capacities gives exactly

$$94,\ 1581,\ 14,\ 254$$

labelled cases in the sectors $(14, 0), (14, 1), (15, 0), (15, 1)$, for a total of

$$\boxed{1943}$$

dangerous cases.

6.1 Excluding cycle components

Since $z = 0$, every component of F is a path or a cycle. For positive-capacity labels define the same bad-pair graph D_Q used above. The step-two labels on any internal cycle yield a closed walk in D_Q. Independent checking gives: all $b = 15$ cases have empty D_Q; for $b = 14, E = 0$, 62 cases have empty D_Q and 32 have D_Q a forest with total incident quota at most 7; for $b = 14, E = 1$, the corresponding counts are 1015 and 566, with total incident quota at most 9. Thus every surviving cycle length must be divisible by four. Since $y \leq 17$ and C_4, C_8, C_{16} are forbidden, only length 12 could remain, but it would require twelve units of incident quota. This exceeds the bounds 7 and 9. Therefore every dangerous case is path-only.

6.2 Endpoint-budget certificates

Each edge $e \in E(Q)$ corresponds to one degree-one vertex x_e of F, which is an endpoint token. For distinct endpoint tokens e, f and a label word $\mathbf{v} = (v_1, \ldots, v_r)$, form a single-path local lift by adding

$$x_e u_1 u_2 \cdots u_r x_f$$

and joining each u_i to its label $v_i \in B$. The word is *locally admissible* if each label occurs at most its capacity t_v and the local lift contains no C_4, C_8, C_{16}.

This local family is deliberately larger than the true lift family: it ignores global quota compatibility, cross-component cycles, perfect-matching feasibility beyond the chosen endpoints, component triggers, and longer power-of-two cycles. Hence every genuine path component is locally admissible.

Lemma 7 (Endpoint-budget certificate)**.** *For each of the* 1943 *dangerous labelled cases there exists*

$$\lambda : E(Q) \to \{0, 1, 2\}$$

such that every locally admissible path with endpoints e, f and r internal vertices satisfies

$$r \leq \lambda(e) + \lambda(f), \tag{12}$$

and

$$\sum_{e \in E(Q)} \lambda(e) < y. \tag{13}$$

The maximum certificate sums in the four sectors are

(b, E)	$(14,0)$	$(14,1)$	$(15,0)$	$(15,1)$
$\max \sum_e \lambda(e)$	13	15	4	6
y	16	17	16	17.

The certificate is static: it stores only the small integers $\lambda(e)$. One verifier enumerates capacity-feasible label words and detects forbidden cycles through exact alternate-path checks. A second implementation directly enumerates simple cycles of lengths $4, 8, 16$ through each newly added internal vertex. It does not depend on the first implementation's path oracle, the earlier exact-cover state space, or the search used to discover λ. Both implementations verify all 1943 certificate inequalities and recover the same sector maxima. The complete older exact-cover and DAG route is retained as a supplementary cross-check but is not logically needed for the endpoint-budget proof.

Deduction of Theorem 2 from Lemma 7. Every endpoint token appears in exactly one true path component, and every y-vertex lies in the interior of exactly one path. If P has endpoints e_P, f_P and $r(P)$ internal vertices, then (12) gives

$$r(P) \le \lambda(e_P) + \lambda(f_P).$$

Summing over all path components,

$$y = \sum_P r(P) \le \sum_P \big(\lambda(e_P) + \lambda(f_P)\big) = \sum_{e \in E(Q)} \lambda(e) < y,$$

a contradiction. Hence the dangerous sectors (11) are impossible, proving $a \ge 2b+10$ for $b \ge 14$. □

7 Computational verification and reproducibility

The proof-critical computations are supplied separately from this manuscript. The release is organised into two independent result modules.

1. **Universal $+8$ module.** It contains the complete small-b atlas regeneration, the global $d = 5$ and $d = 6$ component search for $b = 4, \dots, 7$, the $b = 8$ branch analysis, the $b = 9, 10$ dependency, static charging certificates, and independent cycle oracles.

2. **$+10$ module.** It contains the fixed-defect data, the 1943 dangerous cases, endpoint-budget certificates, two independently written budget verifiers, near-tight witnesses, and the earlier exact-cover/DAG package as a supplementary route.

Public code and computational artefacts. The source code, finite case data, static certificates, independent verification programs, SHA256 manifests, and reproduction instructions used for the computer-assisted parts of this work are publicly available at

https://github.com/LearnChtholly/-erdos-gyarfas-reproducibility.

The repository contains separate modules for the universal additive $+8$ result and the $b \ge 14$ additive $+10$ result, together with supplementary independent verification routes. For reproducibility, a public manuscript version should be associated with the exact Git commit or tagged release used for that version. The packaged SHA256 manifests identify the proof-critical computational artefacts corresponding to each frozen proof bundle.

The finite computations are used only after the mathematical reduction encoded in the proof package shows that every true minimal counterexample in the exceptional sectors maps into the enumerated or relaxed families. In particular, the endpoint-budget arguments use relaxations in the sound direction:

$$\text{true component} \Longrightarrow \text{enumerated locally admissible component.}$$

This interface is the main mathematical point that should be checked independently of the program implementations.

8 Discussion and status

Theorem 1 would sharpen the currently public cubic-density restrictions on hypothetical minimal counterexamples to an asymptotic two-thirds bound with an explicit positive additive term. Theorem 2 improves that additive term further when the number of degree-at-least-four vertices is at least fourteen. We make no claim here that +8 is the first additive strengthening; targeted literature searches have not located an equivalent public bound, but unindexed, unpublished, or differently parameterized overlap remains possible.

The results remain substantially weaker than a resolution of the Erdős–Gyárfás conjecture: they do not rule out a highly cubic minimal counterexample. Their purpose is to narrow the degree structure that any counterexample must have and to expose finite obstruction patterns that may be useful in subsequent work.

Correctness and priority are separate questions. The current proof packages report no known mathematical gap after multiple AI-assisted adversarial audits and contain multiple independently implemented checks for the most delicate finite steps. These internal checks should not be confused with independent human verification. Dedicated external review of the suppression interface, the completeness of the small-component enumeration, and the finite certificates is still required. Likewise, the absence of an equivalent previously published additive bound has not been certified by a domain expert, so this draft intentionally avoids claims such as "first" or "best possible."

AI assistance and verification status. AI systems, including large language models used as research and coding assistants, played a central role in this project. They contributed to mathematical exploration, the proposal and refinement of proof strategies, generation and refactoring of verification code, design of finite searches and certificates, adversarial review, construction of alternative verification implementations, reruns of proof-critical computations, and substantial drafting and editing of the manuscript and reproducibility documentation. Much of the computer-assisted proof development and verification code in the public repository was therefore produced or revised with AI assistance rather than being independently derived by the human author.

The human author selected the research problem, provided high-level direction, chose which outputs to preserve and release, and is responsible for accurately disclosing the status of the work. At the time of this draft, the human author has *not* independently reconstructed all of the mathematical arguments and has *not* personally reproduced all of the computations in a local environment. The computational claims have instead undergone multiple AI-assisted adversarial reviews, reruns, static certificate checks, and separately implemented verification routes as documented in the public proof packages.

Accordingly, the results are presented here as *AI-assisted, computer-assisted candidate results pending independent human verification.* The public code, certificates, frozen inputs, hashes, and reproduction instructions are provided so that external reviewers can evaluate the claims without

trusting the AI systems that helped generate them. Neither AI output nor the internal AI-assisted checking process is treated as a substitute for independent mathematical review.

References

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