Exact $1 \oplus 6$ Spectral Reduction, Discrete Vacuum Topology, and Transverse Response in a Seven-Field Scalar–Tensor Effective Field Theory

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We study a local scalar–tensor effective field theory with seven real scalar fields, an exact signed-permutation symmetry, and a stationary branch on which all seven fields are nonzero. The bulk action admits an exact radial–angular $1 \oplus 6$ decomposition, closed analytic Einstein-frame masses, an exact rational symmetry-breaking point, and a disconnected set of 2^7 degenerate vacua. We also give a complete classification of the physical inverse roots in the light-radial regime. A separately specified transverse response sector is built from a conserved coexact current and a positive spectral representation. Within a restricted local microscopic class, the pair response is represented by a conserved vector current of the form used in photoproduction. At the rational parameter point considered here, the exact tree-level radial–angular P-wave amplitude has a finite-energy zero, $\sqrt{s_{\text{zero}}}/\sqrt{s_0} = 1.141899082719035$. We state explicitly which results follow from the bulk action and which require a choice of response operators, renormalization prescription, state, or matching convention. The resulting EFT is structurally complete within that specified domain, while its identification with a physical realization and any comparison with data remain independent questions.

I. SCOPE, RELATION TO EARLIER WORK, AND LOGICAL STATUS

The aim of this paper is to isolate the local seven-field mathematical core of the L(6+7) research program and present it in a form that can be checked without relying on the surrounding phenomenological corpus. The derivations below are therefore self-contained. Earlier L(6+7) publications are cited to identify the origin and development of the construction, not as substitutes for the proofs given here.

The program was introduced as an across-scale variational framework and subsequently developed through cosmological, microscopic, temporal, gravitational, and realization-oriented branches [1–17]. The original action and cosmological reduction were developed in Refs. [1, 2, 13, 15, 17]; the microscopic vector-source construction in Ref. [4]; and the treatment of temporal order, stable histories, and admissible realization in Refs. [5–7, 14, 16, 17]. The companion works on late-time cosmology, effective gravitational response, cognitive synchronization, recursive phase structure, Hamilton–Jacobi dynamics, and the Einstein–Rosen branch are Refs. [3, 8–12]. The present paper returns to the common local field-theory core and asks a narrower question: which structural statements follow rigorously from the seven-field action and from a separately stated response prescription?

Definition I.1 (Structural closure)**.** An effective field theory will be called *structurally closed within a specified class* when its action, physical degrees of freedom, observable operators, renormalization and matching conventions, and admissible state/boundary data define a map

$$(\text{model}, \varrho_{\text{st}}, \text{boundary/solution data}) \longmapsto \text{observables} \tag{1}$$

without requiring an additional law to be introduced after comparison with observations.

Definition I.2 (Physical realization)**.** A physical realization is a particular solution, quantum state, set of parameter values, and boundary data within the specified EFT.

Definition I.3 (Empirical test)**.** An empirical test compares predictions obtained from a parameter and matching prescription chosen in advance with independent data. Structural closure by itself does not imply empirical confirmation.

Possible non-uniqueness of a UV completion limits the range over which an EFT can be regarded as universal, but it does not make the low-energy theory mathematically incomplete within its stated domain.

Two logically distinct ingredients are used below. The symmetry, stationary branch, scalar field-space geometry, Hessian, and scalar interaction vertices are derived from the variational bulk action. The microscopic observable

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sector is specified separately through an operator basis, conservation conditions, a renormalization prescription, and a matching convention. In particular, we do not claim that the coexact current is obtained by varying the scalar–tensor bulk action alone.

We reserve ρ for the radial field coordinate, $\rho_{IJ}(s)$ for a spectral density, and $\varrho_{\rm st}$ for a density operator. The coupling $a = n\gamma/\lambda$ is distinct from the FRW scale factor $a_{\rm FRW}$. The quartic coupling is denoted by λ, whereas the Källén function is denoted by $\lambda_{\rm K}$.

II. PARENT VARIATIONAL ACTION AND STATIONARY BRANCH

The local seven-field action used here is the scalar–tensor sector that underlies the earlier across-scale and later consolidated L(6+7) formulations [1, 13, 15, 17]. We write it explicitly so that no result in this paper depends on an external definition.

We consider

$$S = \int d^4x\,\sqrt{-g}\left[\frac{M_0^2}{2}F(S_X)R - \frac{1}{2}\sum_{A=1}^{n}(\nabla X_A)^2 - V(X)\right], \tag{2}$$

with

$$F(S_X) = 1 + \bar{\xi}S_X, \qquad S_X = \sum_{A=1}^{n} X_A^2, \tag{3}$$

and

$$V(X) = \frac{\lambda}{4}\sum_{A=1}^{n}(X_A^2 - v^2)^2 + \frac{\gamma}{4}\left(\sum_{A=1}^{n}X_A^2\right)^2. \tag{4}$$

The second quartic contains genuine cross-mode interactions,

$$\left(\sum_A X_A^2\right)^2 = \sum_A X_A^4 + 2\sum_{A<B}X_A^2X_B^2. \tag{5}$$

At the level of internal quartic invariants, Eq. (4) is a member of the well-studied cubic-anisotropy (hypercubic) class, in which an $O(n)$-symmetric quartic invariant is combined with $\sum_A X_A^4$; here that structure is augmented by the nonminimal curvature coupling and the symmetry-breaking scale v [22, 23]. We introduce

$$a = \frac{n\gamma}{\lambda}, \qquad b = n\bar{\xi}v^2, \qquad \mu = \frac{v^2}{M_0^2}, \qquad L = 1 + a + b, \qquad E = (1+b)^2 + a. \tag{6}$$

For the permutation-symmetric configuration $X_A = x$, $y = x^2$, the stationary de Sitter equations are

$$3M_0^2F_\star H_{J,\star}^2 = V_\star, \qquad V_{,A} - \frac{M_0^2}{2}F_{,A}R = 0, \qquad R = 12H_{J,\star}^2. \tag{7}$$

Theorem II.1 (Exact full-support stationary point). *For $L \neq 0$, the symmetric full-support branch obeys*

$$\boxed{\frac{x_\star^2}{v^2} = \frac{1+b}{L}}, \qquad \boxed{F_\star = \frac{E}{L}}. \tag{8}$$

Proof. Substitution of $X_A = x$ into Eqs. (2)–(4) makes all n scalar equations identical. Eliminating $H_{J,\star}^2$ between the scalar and Friedmann equations gives a linear equation for $y = x^2$,

$$\frac{y_\star}{v^2} = \frac{1+b}{1+a+b}.$$

Using $F = 1 + n\bar{\xi}y$ then gives $F_\star = E/L$. □

A natural physical sector is

$$\lambda > 0, \qquad a > 0, \qquad b > -1, \qquad \mu > 0, \qquad F_\star > 0, \tag{9}$$

which ensures a positive quartic scale, nonnegative full-support radius, and positive effective Planck mass.

III. EINSTEIN-FRAME GEOMETRY AND THE $1 \oplus (n-1)$ SPECTRUM

Under

$$g^E_{\mu\nu} = F\, g^J_{\mu\nu}, \tag{10}$$

the scalar sector has target-space metric

$$G^E_{AB} = \frac{\delta_{AB}}{F} + \frac{6M_0^2\bar{\xi}^2}{F^2} X_A X_B, \qquad U(X) = \frac{V(X)}{F(X)^2}. \tag{11}$$

This type of multi-scalar Einstein-frame field-space geometry is standard in nonminimally coupled scalar–tensor theories [18, 19].

Lemma III.1 (Permutation reduction of the Hessian)**.** *At a full-support symmetric point, every permutation-invariant quadratic form has the form*

$$\mathcal{H}_{AB} = \alpha\, \delta_{AB} + \beta\, \mathbf{1}_A \mathbf{1}_B. \tag{12}$$

Hence there is one singlet eigenvector

$$e_r = \frac{1}{\sqrt{n}}(1, \dots, 1) \tag{13}$$

and $n-1$ degenerate eigenvectors orthogonal to e_r.

Proof. A matrix commuting with all permutation matrices of the natural representation of S_n lies in the commutant spanned by I and $J = \mathbf{1}\mathbf{1}^T$. Thus

$$\mathbf{n} = \mathbf{1} \oplus (\mathbf{n} - \mathbf{1}). \tag{14}$$

□

To make the normalization explicit, introduce

$$\Delta_F = \frac{6b^2(1+b)}{n\mu}, \qquad \mathcal{D} = E + \Delta_F. \tag{15}$$

Let $P_r = e_r e_r^T$ and $P_a = I - P_r$ denote the singlet and standard projectors. At the stationary point, the Einstein-frame kinetic metric and the covariant Hessian of U reduce to

$$G^E_\star = g_r P_r + g_a P_a, \qquad g_r = \frac{L\mathcal{D}}{E^2}, \qquad g_a = \frac{L}{E}, \tag{16}$$

$$\nabla\nabla U\big|_\star = h_r P_r + h_a P_a, \qquad h_r = 2\lambda v^2 \frac{(1+b)L^3}{E^3}, \qquad h_a = 2\lambda v^2 \frac{(1+b)L}{E^2}. \tag{17}$$

The canonical masses are the generalized eigenvalues of the pair $(\nabla\nabla U, G^E)$, and therefore

$$m_a^2 = \frac{h_a}{g_a} = 2\lambda v^2 \frac{1+b}{E}, \tag{18}$$

$$m_r^2 = \frac{h_r}{g_r} = 2\lambda v^2 \frac{(1+b)L^2}{E\mathcal{D}}. \tag{19}$$

This also shows directly why all $n-1$ angular modes remain degenerate: both quadratic forms are proportional to P_a on the standard representation.

For completeness, the stationary potential and the Jordan-frame Hubble scale are

$$V_\star = \frac{a\lambda n v^4 E}{4L^2}, \qquad H^2_{J,\star} = \frac{a\lambda n\mu v^2}{12L}. \tag{20}$$

Combining these expressions with Eqs. (18)–(19) gives

$$R_a^{\rm alg} \equiv \frac{m_a^2}{H^2_{J,\star}} = \frac{24(1+b)L}{na\mu E}, \tag{21}$$

$$R_r^{\rm alg} \equiv \frac{m_r^2}{H^2_{J,\star}} = \frac{24(1+b)L^3}{na\mu E\mathcal{D}}, \tag{22}$$

$$q^2 \equiv \frac{m_r^2}{m_a^2} = \frac{L^2}{\mathcal{D}}. \tag{23}$$

IV. FRAME NORMALIZATION

At a stationary point with constant $F_\star$,

$$dt_E = \sqrt{F_\star}\, dt_J, \qquad a_E = \sqrt{F_\star}\, a_J, \tag{24}$$

so

$$\boxed{H^2_{E,\star} = \frac{H^2_{J,\star}}{F_\star}}. \tag{25}$$

Theorem IV.1 (Frame-correct relaxation)**.** *For canonically normalized Einstein-frame masses define*

$$R_i^{\rm alg} \equiv \frac{m^2_{i,E}}{H^2_{J,\star}}, \qquad \widehat{R}_i \equiv \frac{m^2_{i,E}}{H^2_{E,\star}}. \tag{26}$$

Then

$$\boxed{\widehat{R}_i = F_\star R_i^{\rm alg}}. \tag{27}$$

The canonical fixed-point relaxation equation is

$$\chi_i'' + 3\chi_i' + \widehat{R}_i \chi_i = 0, \tag{28}$$

where the prime denotes d/dN_E.

Proof. Equation (27) follows directly from Eq. (25). Since $N_E = N_J + \frac{1}{2}\ln F_\star$ at constant $F_\star$, $d/dN_E = d/dN_J$ at the fixed point, and the canonical Einstein-frame mass term is normalized by H_E^2. □

Corollary IV.2 (Frame-invariant mass ratio)**.**

$$\boxed{q^2 = \frac{\widehat{R}_r}{\widehat{R}_a} = \frac{R_r^{\rm alg}}{R_a^{\rm alg}} = \frac{m_r^2}{m_a^2}}. \tag{29}$$

V. EXACT RATIONAL PARAMETER POINT

For

$$n = 7, \qquad a = \frac{32}{33}, \qquad b = -\frac{7}{11}, \tag{30}$$

one obtains

$$L = \frac{4}{3}, \qquad E = \frac{400}{363}. \tag{31}$$

On the equal-magnitude orbit $X_A = v\rho$,

$$F = 1 - \frac{7}{11}\rho^2, \tag{32}$$

and

$$V = \frac{\lambda n v^4}{4}\left(\frac{65}{33}\rho^4 - 2\rho^2 + 1\right). \tag{33}$$

Eliminating ρ gives

$$W(F) = \frac{715F^2 - 968F + 400}{147}, \qquad U(F) = \frac{\lambda n v^4}{4}\frac{W(F)}{F^2}. \tag{34}$$

Theorem V.1 (Exact broken radial minimum)**.** *For Eq.* (34)*,*

$$\frac{d}{dF}\left[\frac{U}{\lambda n v^4/4}\right] = \frac{8(121F-100)}{147F^3}. \tag{35}$$

The unique positive stationary point is

$$\boxed{F_\star = \frac{100}{121}}, \tag{36}$$

with

$$\boxed{\rho_\star^2 = \frac{33}{121}, \qquad \frac{R_\star}{v} = \sqrt{\frac{21}{11}}, \qquad U_\star = \frac{77}{50}\lambda v^4}. \tag{37}$$

Proof. The stationary condition is $121F - 100 = 0$. Then

$$\rho_\star^2 = \frac{11}{7}\left(1 - \frac{100}{121}\right) = \frac{33}{121}.$$

For $n = 7$, $R^2 = \sum_A X_A^2 = 7v^2\rho^2$, yielding $R_\star/v = \sqrt{21/11}$. Direct substitution into Eq. (34) gives $U_\star = (77/50)\lambda v^4$. □

VI. SIGNED-PERMUTATION VACUUM TOPOLOGY

At fixed

$$S_X = \sum_A X_A^2, \tag{38}$$

the orientation-dependent part of the potential is proportional to $\sum_A X_A^4$.

Lemma VI.1 (Orientation minimization)**.** *For real X_A,*

$$\sum_{A=1}^{n} X_A^4 \geq \frac{1}{n}\left(\sum_{A=1}^{n} X_A^2\right)^2, \tag{39}$$

with equality if and only if

$$|X_1| = \cdots = |X_n|. \tag{40}$$

Proof. This is the Cauchy–Schwarz inequality applied to $(X_1^2, \ldots, X_n^2)$ and $(1, \ldots, 1)$. □

For $n = 7$, the action-derived sign rays are

$$X_A = s_A r, \qquad s_A \in \{+1, -1\}. \tag{41}$$

Theorem VI.2 (Disconnected broken vacuum set)**.** *At the exact signed-permutation-symmetric broken minimum,*

$$\mathcal{M}_{\mathrm{vac}} = \{X_A = s_A v \rho_\star; \quad s_A = \pm 1\} \tag{42}$$

contains exactly

$$\boxed{2^7 = 128} \tag{43}$$

isolated degenerate points.

Corollary VI.3 (Conditional domain-wall statement)**.** *If two asymptotic spatial regions select distinct components of $\mathcal{M}_{\mathrm{vac}}$, any continuous field configuration interpolating between them must leave the vacuum set. If the action-derived barrier is positive, the interface has nonzero surface energy and is a domain wall.*

The general existence of 2^N-vacuum field-theory landscapes is not novel by itself [24]; the result established here is the exact realization of this topology by the specific action and parameter choice studied below.

VII. FRAME-COVARIANT CLASSIFICATION OF PHYSICAL ROOTS

Let

$$f = F_\star, \qquad q^2 = \frac{\widehat{R}_r}{\widehat{R}_a}. \tag{44}$$

Using $R_a^{\text{alg}} = \widehat{R}_a/f$, the inverse equation is

$$q^2\left[f + \frac{\widehat{R}_a}{4}b(1+b-f)\right] = \frac{b(1+b)}{f-1}. \tag{45}$$

At the rational reference point used below,

$$f = \frac{100}{121}, \qquad b = -\frac{7}{11}, \qquad \widehat{R}_a = 29.93967042530, \qquad q^2 = 0.4399177206603,$$

the two sides of Eq. (45) evaluate independently to

$$q^2\left[f + \frac{\widehat{R}_a}{4}b(1+b-f)\right] = 1.3333333333333 = \frac{b(1+b)}{f-1}. \tag{46}$$

This explicit substitution is included to make the frame normalization and the factor of 1/4 in the inverse relation unambiguous.

After selecting a physical root b,

$$a = \frac{(1+b)(1+b-f)}{f-1}, \tag{47}$$

$$\mu = \frac{24(f-1)}{n\widehat{R}_a(1+b-f)}. \tag{48}$$

Theorem VII.1 (Light-radial root classification)**.** *Assume* $0 < q^2 < 1$*. Then:*

1. *for* $0 < f < 1$*, exactly one physical root exists;*
2. *for* $f = 1$*, no physical light-radial root exists;*
3. *for* $f > 1$*, exactly one physical root exists if and only if*

$$\boxed{\widehat{R}_a > \frac{4}{q^2(f-1)}} \tag{49}$$

or, equivalently,

$$\boxed{\widehat{R}_r > \frac{4}{f-1}}. \tag{50}$$

Proof. At fixed $f > 0$ and $\widehat{R}_a > 0$, define

$$Q(b) = \frac{\dfrac{b(1+b)}{f-1}}{f + \dfrac{\widehat{R}_a}{4}b(1+b-f)}. \tag{51}$$

The denominator is strictly positive on both physical branches. Direct differentiation gives

$$Q'(b) = -\frac{4f\left(\widehat{R}_a b^2 - 8b - 4\right)}{(f-1)\left(\widehat{R}_a b^2 - \widehat{R}_a f b + \widehat{R}_a b + 4f\right)^2}. \tag{52}$$

The two stationary candidates are

$$b_\pm = \frac{4 \pm 2\sqrt{4+\widehat{R}_a}}{\widehat{R}_a}, \qquad -1 < b_- < 0 < b_+. \tag{53}$$

For $0 < f < 1$, the physical interval is $-1 < b < f-1 < 0$, with

$$Q(-1^+) = 0, \qquad Q((f-1)^-) = 1. \tag{54}$$

The only possible interior stationary point is b_-, which is a maximum. If it lies outside the physical interval, Q is monotone from 0 to 1. If it lies inside, Q first rises above the endpoint level 1 and then decreases to 1. Hence every level $0 < q^2 < 1$ is crossed exactly once.

For $f > 1$, the physical interval is $b > f - 1 > 0$, with

$$Q((f-1)^+) = 1, \qquad \lim_{b\to\infty} Q(b) = q_\infty^2 = \frac{4}{\widehat{R}_a(f-1)}. \tag{55}$$

There is at most one positive stationary point, b_+, and it is a maximum. Therefore a level $0 < q^2 < 1$ is crossed exactly once if and only if $q_\infty^2 < q^2$, which is Eq. (50).

It remains to treat $f = 1$ without using the singular inverse parametrization. From $f = E/L = 1$ one finds

$$(1+b)^2 + a = 1 + a + b \quad \Longrightarrow \quad b(b+1) = 0.$$

The physical condition $b > -1$ leaves $b = 0$. Equation (23) then gives $q^2 = L = 1+a > 1$ because $a > 0$, contradicting the assumed light-radial condition $q^2 < 1$. Hence no physical light-radial root exists at $f = 1$. □

VIII. SYMMETRIC COSMOLOGICAL BRANCH AND CHANGE OF STABILITY

This subsection is a cosmological specialization of the scalar–tensor sector. In addition to Eq. (2), we include minimally coupled pressureless matter and radiation only as background sources; they do not alter the scalar field-space geometry or the stationary spectrum derived above. With $N = \ln a_{\mathrm{FRW}}$, the symmetric configuration $\rho = 0$ satisfies

$$F = 1, \qquad F_{,\rho} = 0, \qquad V_{,\rho} = 0, \tag{56}$$

and is therefore an exact homogeneous solution. The radial linearization is

$$\kappa\rho'' + \kappa(3+x)\rho' + C(N)\rho = 0, \tag{57}$$

where

$$x = \frac{d\ln H}{dN}, \qquad \kappa = 7\mu, \qquad h^2 = M + R + D. \tag{58}$$

Here $\kappa = 7\mu$ is the equal-component radial kinetic coefficient for the $n = 7$ model considered below, and

$$M = \Omega_m e^{-3N}, \qquad R = \Omega_r e^{-4N}. \tag{59}$$

Here $D > 0$ denotes the constant vacuum-energy contribution in the same dimensionless background normalization, so that $h^2 = M + R + D$. This notation is the one used in the earlier cosmological reduction [2, 8, 13]. The mass term obtained by linearizing the action is

$$Ch^2 = -3bM - 12(1+b)D. \tag{60}$$

Proposition VIII.1 (Exact stability crossing)**.** *The radiation contribution cancels from the numerator in Eq.* (60)*, and the crossing is*

$$\boxed{\frac{M_c}{D} = -\frac{4(1+b)}{b}}. \tag{61}$$

For $b = -7/11$*,*

$$\boxed{\frac{M_c}{D} = \frac{16}{7}}. \tag{62}$$

Thus the symmetry-breaking transition on this branch is generated by the sign change of an action-defined effective mass; no additional phenomenological phase-transition operator is introduced.

IX. CONSERVED CURRENT AND POSITIVE SPECTRAL REPRESENTATION

Consider the coexact current

$$J_7^\mu = \nabla_\nu M^{\nu\mu}, \qquad M^{\mu\nu} = -M^{\nu\mu}. \tag{63}$$

Theorem IX.1 (Covariant conservation)**.** *For the Levi-Civita connection,*

$$\boxed{\nabla_\mu J_7^\mu = 0} \tag{64}$$

identically, provided the renormalized operator basis preserves the antisymmetric two-form structure.

Proof.

$$\nabla_\mu J_7^\mu = \nabla_\mu \nabla_\nu M^{\nu\mu} = \frac{1}{2}[\nabla_\mu, \nabla_\nu] M^{\nu\mu}. \tag{65}$$

The commutator produces contractions of the form $R_{\lambda\mu} M^{\lambda\mu}$, which vanish because the Ricci tensor is symmetric and $M^{\mu\nu}$ is antisymmetric. □

A local scalar representative can be organized as

$$\mathcal{O}_7^\mu = \nabla_\nu \left[\mathcal{B}_{AB}(X) \nabla^\nu X^A \nabla^\mu X^B\right]_R, \qquad \mathcal{B}_{AB} = -\mathcal{B}_{BA}, \qquad [\mathcal{O}_7] = 5, \tag{66}$$

with a dimension-three contribution

$$J^\mu_{7,\mathrm{phys}} = \frac{c_7}{\Lambda_7^2} \mathcal{O}_7^\mu, \tag{67}$$

where c_7 is a dimensionless Wilson coefficient and Λ_7 is the corresponding EFT suppression scale.

For the spectral analysis we now restrict to a translationally invariant local EFT patch in which the usual momentum-space spectral representation is applicable. For a set of transverse currents J_I^μ, define

$$\Pi_{IJ}^{\mu\nu}(q) = i \int d^4x\, e^{iqx} \langle \Omega | T J_I^\mu(x) J_J^\nu(0) | \Omega \rangle. \tag{68}$$

After contact/subtraction terms, the transverse scalar sector has spectral density $\rho_{IJ}(s)$.

Theorem IX.2 (Positive spectral matrix)**.** *For every complex vector c_I,*

$$\boxed{c_I^* \rho_{IJ}(s) c_J \geq 0} \tag{69}$$

as a distribution on the physical support.

Proof. Insertion of a complete set of physical intermediate states gives the Gram representation

$$\rho_{IJ}(s) \sim \sum_n \delta(s - m_n^2) \langle \Omega | J_I | n \rangle \langle n | J_J | \Omega \rangle, \tag{70}$$

and contraction with c_I gives a sum of absolute squares. □

This is the standard Källén–Lehmann/Gram-positivity structure for current two-point functions [20]. For the coexact contribution, canonical UV counting gives

$$\rho_{77}(s) \sim \frac{c_7^2}{\Lambda_7^4} s^2 [\log s + \cdots], \tag{71}$$

so the moments

$$D_k(Q^2) = \int_0^\infty \frac{\rho(s)\, ds}{(s + Q^2)^{k+1}} \tag{72}$$

are absolutely UV convergent for

$$\boxed{k \geq 3} \tag{73}$$

in the generic mixed-current sector.

X. RESTRICTED TRANSVERSE VECTOR-CURRENT RESPONSE

The microscopic construction used here is the local vector-current sector developed in the photoproduction branch of the L(6+7) program [4]. The statement below is deliberately restricted to the explicitly listed microscopic class.

Theorem X.1 (Restricted vector-current representation)**.** *Let $\mathcal{M}_{\rm min}$ be the microscopic class satisfying:*

1. *local Lorentz-covariant QFT in four dimensions;*
2. *parity-even, vector-like microscopic pair response;*
3. *a Dirac pair field ψ and a derivative-free renormalizable linear coupling to a vector carrier in the minimal representative;*
4. *exact current conservation and the Ward identity;*
5. *no additional fundamental response fields and no nonlocal channel-selection rule;*
6. *a nontrivial connected transverse spectral measure.*

Then the quantum transverse pair-response closure in $\mathcal{M}_{\rm min}$ factors through a conserved photoproduction/vector-current layer, modulo improvement and equations-of-motion equivalent operators.

Proof. A prescribed c-number current has no operator-valued fluctuations or connected spectral measure of its own. Premise 6 therefore requires the pair-response carrier to be an operator in the quantum theory rather than merely a classical source.

The derivative-free Dirac bilinears are

$$\bar\psi\psi, \quad \bar\psi i\gamma_5\psi, \quad \bar\psi\gamma^\mu\psi, \quad \bar\psi\gamma^\mu\gamma_5\psi, \quad \bar\psi\sigma^{\mu\nu}\psi. \tag{74}$$

Under premises 1–3, the unique dimension-three bilinear with one free Lorentz-vector index, parity-even vector transformation, and a derivative-free renormalizable linear coupling to a vector carrier is

$$J^\mu_{\rm pair} = \bar\psi\gamma^\mu T\psi, \tag{75}$$

up to improvement/EOM equivalence. For a fermion mass matrix M_ψ satisfying $[M_\psi, T] = 0$, together with symmetry-preserving interactions, the current is conserved and the Ward identity enforces transversality. Pair states then produce a nontrivial positive continuum. Thus, within the stated microscopic class, every minimal derivative-free pair-response representative is equivalent to a conserved Dirac vector current up to improvement and equations-of-motion terms. Coupling this current to the vector carrier gives the photoproduction/vector-current realization used here. □

Remark X.2. This is a theorem *within* $\mathcal{M}_{\rm min}$, not a uniqueness theorem over arbitrary nonlocal, higher-form, or additional-field UV completions. It also should not be identified with an unrestricted phenomenological vector-meson-dominance ansatz; the latter can fail in some channels, in particular for heavy vector mesons [25].

In the representative below, $J^\mu_{\rm eff}$ denotes the conserved vector source selected within this class: it contains the Dirac Noether current and, when retained in the microscopic matching, the coexact contribution $J^\mu_{7,\rm phys}$. Any prescribed classical external source is specified separately.

A minimal representative may be written as

$$\begin{aligned}\mathcal{L}_{\rm micro} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu - \frac{e}{2f_V}F_{\mu\nu}V^{\mu\nu} \\ & + \bar\psi(i\gamma^\mu\partial_\mu - m)\psi - J^\mu_{\rm eff}V_\mu. \end{aligned}\tag{76}$$

Channel-resolved numerical values of f_V must be supplied by an independent matching calculation; they are not adjustable coefficients to be chosen after comparison with data.

XI. RADIAL–ANGULAR THRESHOLD AND EXACT TREE P-WAVE

The scattering calculation in this section is a local tangent-space result around the stationary Einstein-frame background. It uses the adiabatic particle interpretation of the scalar fluctuations in that neighborhood; no global de Sitter S-matrix is assumed.

The same geometry that determines the masses also fixes the three invariants needed for the tree-level P-wave. In Riemann normal coordinates at the stationary point, define the radial–angular sectional curvature and the two nonvanishing cubic covariant derivatives of the potential by

$$K_{ra} \equiv R_{rara} = \frac{bL}{nv^2\mathcal{D}^2}\left[\mathcal{D} + \left(1+\frac{6b}{n\mu}\right)E\right], \tag{77}$$

$$g_{raa} \equiv U_{;(raa)} = \frac{2\lambda v\sqrt{1+b}\,L}{E\sqrt{n}\,\mathcal{D}^{3/2}}\left[3E + a(a+2b+1) + 2\Delta_F\right], \tag{78}$$

$$g_{rrr} \equiv U_{;(rrr)} = \frac{6\lambda v\sqrt{1+b}\,L^2}{E\sqrt{n}\,\mathcal{D}^{5/2}}\left[E(a-2b^2-b+1) - 2b(1+b)\Delta_F\right]. \tag{79}$$

For $b=0$ the scalar field space is flat and $K_{ra}=0$, providing a useful check on Eq. (77). Permutation symmetry also gives $g_{rra}=0$ for a single standard-representation index.

Let $\Lambda_{rraa} = U_{;(rraa)}$ denote the symmetrized quartic covariant derivative. The complete local tree amplitude for $r+a\to r+a$ is

$$\begin{aligned}\mathcal{M}_{ra}(s,t,u) = &-\Lambda_{rraa} + K_{ra}\left[t - \frac{2(m_r^2+m_a^2)}{3}\right]\\ &-\frac{g_{raa}^2}{s-m_a^2} - \frac{g_{rrr}g_{raa}}{t-m_r^2} - \frac{g_{raa}^2}{u-m_a^2}.\end{aligned} \tag{80}$$

The contact term and the s-channel term are independent of the scattering angle and therefore drop out of the $\ell=1$ projection.

In the broken full-support sector the first scalar coexact two-particle channel has

$$s_0 = (m_r+m_a)^2. \tag{81}$$

For unequal masses, P-wave phase space gives

$$\rho_{ra}(s) \propto \frac{\lambda_{\mathrm{K}}(s,m_r^2,m_a^2)^{3/2}}{s}\theta(s-s_0), \tag{82}$$

where

$$\lambda_{\mathrm{K}}(s,m_r^2,m_a^2) = [s-(m_r+m_a)^2][s-(m_r-m_a)^2]. \tag{83}$$

For elastic unequal-mass kinematics,

$$p^2(s) = \frac{\lambda_{\mathrm{K}}(s,m_r^2,m_a^2)}{4s}, \qquad t = -2p^2(1-z), \qquad u = 2(m_r^2+m_a^2) - s - t, \tag{84}$$

and we use

$$\mathcal{M}(s,z) = 16\pi\sum_{\ell\geq 0}(2\ell+1)t_\ell(s)P_\ell(z), \qquad t_\ell(s) = \frac{1}{32\pi}\int_{-1}^{1} dz\, P_\ell(z)\mathcal{M}(s,z). \tag{85}$$

Define

$$Q_1(x) = \frac{x}{2}\ln\frac{x+1}{x-1} - 1, \quad x_t = 1+\frac{m_r^2}{2p^2}, \quad x_u = \frac{s-2m_r^2-m_a^2-2p^2}{2p^2}. \tag{86}$$

On the physical sheet in the light-radial regime $m_r < 2m_a$, the exact tree-level amplitude obtained from the action is

$$\boxed{t_1^{\mathrm{tree}}(s) = \frac{1}{32\pi}\left[\frac{4}{3}K_{ra}p^2 + \frac{g_{rrr}g_{raa}}{p^2}Q_1(x_t) - \frac{g_{raa}^2}{p^2}Q_1(x_u)\right]}. \tag{87}$$

Equation (87) follows by projecting Eq. (80). Since $Q_1(x) = 1/(3x^2) + O(x^{-4})$,

$$t_1^{\rm tree}(s) = \frac{p^2}{24\pi} B_{ra} + O(p^4), \tag{88}$$

where

$$B_{ra} = K_{ra} + \frac{g_{rrr} g_{raa}}{m_r^4} - \frac{g_{raa}^2}{m_r^2(2m_a - m_r)^2}. \tag{89}$$

Substitution of the rational parameter point of Sec. V into Eqs. (77)–(79) gives

$$K_{ra} v^2 = 0.06571081123650, \tag{90}$$

$$\frac{g_{raa}}{\lambda v} = 0.66945620600207, \tag{91}$$

$$\frac{g_{rrr}}{\lambda v} = 0.22442945768336, \tag{92}$$

and

$$\boxed{B_{ra} v^2 = 0.53911206383543 > 0}. \tag{93}$$

The first generic scalar inelastic threshold is

$$\sqrt{s_{\rm inel}} = \min(2m_r + m_a,\, 3m_a). \tag{94}$$

In the light-radial regime $m_r < m_a$, this reduces to $\sqrt{s_{\rm inel}} = 2m_r + m_a$. Using $H_{J,\star}$ as the Jordan-frame reference scale,

$$\frac{m_r}{H_{J,\star}} = 3.99210468308278, \qquad \frac{m_a}{H_{J,\star}} = 6.01888704119061, \tag{95}$$

so

$$\frac{\sqrt{s_0}}{H_{J,\star}} = 10.0109917242734, \qquad \frac{\sqrt{s_{\rm inel}}}{H_{J,\star}} = 14.0030964073562. \tag{96}$$

Solving Eq. (87) at this parameter point gives the finite-energy root

$$\boxed{\frac{\sqrt{s_{\rm zero}}}{\sqrt{s_0}} = 1.141899082719035}. \tag{97}$$

The number in Eq. (97) is a dimensionless ratio. Identifying it with a particular experimental energy or resonance requires an independently fixed physical scale and a channel-specific matching prescription; no such identification is assumed in deriving the result reported here.

Remark XI.1 (Tree/1PI boundary). Equation (97) is a tree-level prediction of the specified action. It does not imply that the full 1PI or nonperturbative amplitude must retain a zero at the same location. Curved scalar field space generically requires higher-derivative counterterms in the EFT loop expansion, whose finite renormalized Wilson coefficients are not fixed by the two-derivative truncation alone [19, 21]. A shifted, split, or removed zero after dressing is therefore a possible dynamical outcome. Such a result would be interpreted within the renormalized theory rather than by changing the tree-level parameters after the calculation.

XII. CANONICAL EVOLUTION AND RESPONSE/STATE SEPARATION

Choose a reference e-fold B on the symmetric branch. For one Fourier mode the canonical momentum can be written as

$$p_k = a_{\rm FRW}^3 \kappa H\, \rho_k', \qquad s(N) = a_{\rm FRW}^3 \kappa H. \tag{98}$$

We suppress the mode label k below and write

$$u = (\rho, \rho')^T, \qquad u(N) = T_u(N, B) u(B). \tag{99}$$

then

$$z = (\rho, p)^T = S(N)u, \qquad S(N) = \mathrm{diag}(1, s(N)), \tag{100}$$

and

$$z(N) = T_{\mathrm{can}}(N, B)z(B), \qquad T_{\mathrm{can}} = S(N)T_u(N, B)S(B)^{-1}. \tag{101}$$

Theorem XII.1 (Symplectic transfer)**.** *Let*

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

be the canonical symplectic form. For exact linearized Hamiltonian dynamics,

$$\boxed{\det T_{\mathrm{can}} = 1}, \qquad \boxed{T_{\mathrm{can}}^T J T_{\mathrm{can}} = J}. \tag{102}$$

Consequently, if Σ_B is the phase-space covariance at the reference e-fold B,

$$\Sigma(N) = T_{\mathrm{can}} \Sigma_B T_{\mathrm{can}}^T, \qquad \det \Sigma(N) = \det \Sigma_B. \tag{103}$$

Theorem XII.2 (Retarded/Hadamard separation)**.** *After fixing the action, background, and canonical normalization:*

1. *the retarded/commutator kernel G_R is determined by the transfer operator and does not depend on occupation or squeezing;*

2. *for a centered Gaussian state, the symmetrized field kernel*

$$G_{\mathrm{H}}(N_1, N_2) \equiv \frac{1}{2}\langle\{\rho(N_1), \rho(N_2)\}\rangle$$

is

$$G_{\mathrm{H}}(N_1, N_2) = e_q^T T_{\mathrm{can}}(N_1, B)\, \Sigma_B\, T_{\mathrm{can}}(N_2, B)^T e_q, \qquad e_q = (1, 0)^T, \tag{104}$$

and therefore depends on the physical boundary state Σ_B.

Thus uncertainty in the initial density operator is uncertainty in the physical realization, not uncertainty in the equations of motion.

XIII. CLOSURE WITHIN THE SPECIFIED EFT

For the purpose of the following statement, let $\mathcal{T}_{L67}$ denote the complete low-energy model definition consisting of:

1. the bulk variational action, Eq. (2);
2. the Einstein-frame geometry, Eq. (11);
3. the physical all-components-nonzero branch and the $1 \oplus 6$ reduction;
4. the exact B_7/S_7 symmetry class;
5. a conserved transverse response-operator basis specified before comparison with data;
6. the positive spectral representation and a fixed renormalization prescription;
7. the microscopic carrier and matching convention used in the selected response sector;
8. the stated cross-scale reduction rules; and
9. the requirement that the preceding ingredients are not altered in response to the outcome of a validation test.

Theorem XIII.1 (Closure within the specified model class)**.** *With items 1–9 specified, and within the stated local EFT domain, $\mathcal{T}_{L67}$ is structurally closed in the sense defined in Sec. I: the map*

$$(\mathcal{T}_{L67}, \varrho_{\mathrm{st}}, \textit{boundary/solution data}) \longmapsto \textit{observables} \tag{105}$$

does not require an additional independent bulk or response law once a physical state and boundary data have been chosen.

Proof. The bulk variational principle fixes the equations of motion and their constraints. The symmetry, field-space geometry, and Hessian determine the stationary physical degrees of freedom and the tree-level scalar vertices. Items 5–7 specify the observable operators and the prescription used to construct renormalized response functions. Given a state ϱ_{st} and boundary/solution data, the retarded dynamics and state-dependent correlation functions are then determined by those choices. The last two items ensure that a disagreement with a later observation is not absorbed by introducing an additional law into the same model. The state and solution are therefore inputs to a specified theory, rather than new dynamical laws. □

Remark XIII.2. This theorem concerns completeness of a specified low-energy EFT. It is not a uniqueness theorem for UV completion. In particular, we do not claim a unique photoproduction realization outside the microscopic class stated above, nor do we claim that finite higher-derivative Wilson coefficients are fixed by the two-derivative action.

XIV. REFERENCE PARAMETER POINT AND PROSPECTIVE TESTS

For the numerical examples and the reproducibility calculation we use

$$n = 7, \qquad a = \frac{32}{33}, \qquad b = -\frac{7}{11}, \qquad \mu = 0.04294350163012621. \tag{106}$$

This gives

$$F_\star = 0.826446280991735 = \frac{100}{121}, \tag{107}$$

$$R_r^{\mathrm{alg}} = 15.93689980069, \tag{108}$$

$$R_a^{\mathrm{alg}} = 36.22700121461, \tag{109}$$

$$\widehat{R}_r = 13.17099157082, \tag{110}$$

$$\widehat{R}_a = 29.93967042530, \tag{111}$$

$$q^2 = 0.4399177206603. \tag{112}$$

These numbers define a reference realization of the theory. Their agreement with properties of our Universe is not assumed anywhere in the structural results and must be assessed independently.

For any future comparison with validation data, the model version, frame conventions, physical-root prescription, microscopic matching convention, state prescription, numerical procedure, uncertainty treatment, and decision criteria should be fixed before the validation data are inspected. If any of these choices is changed in response to the result, the modified construction is a new realization and requires a new test.

Direct ways to challenge the present construction include an irreducible splitting of the six angular modes once known symmetry breaking is removed; violation of the frame relation $\widehat{R}_i = F_\star R_i^{\mathrm{alg}}$; a counterexample to the physical-root theorem within its assumptions; a negative eigenvalue of the properly normalized physical spectral matrix; an independently established lower support or different threshold exponent for the stated $r + a$ P-wave channel; or a calculation from the same action that fails to reproduce the tree-level amplitude in Eq. (87).

XV. DISCUSSION AND CONCLUSIONS

The construction developed here follows a simple logical sequence:

$$\begin{aligned} &\text{bulk action} \to \text{stationary } 1 \oplus 6 \text{ reduction} \to \text{specified response sector} \\ &\qquad \to \text{physical state and solution} \to \text{independent empirical test.} \end{aligned} \tag{113}$$

The first three steps define the theory studied in this paper; the last two determine whether a particular realization describes nature.

The strongest model-specific results are the exact full-support stationary solution, the explicitly normalized $1 \oplus 6$ spectrum, the rational symmetry-breaking point, the classification of physical inverse roots, and the radial–angular tree-level P-wave amplitude derived from the same field-space invariants, including its finite-energy zero. The disconnected 128-point vacuum set, the coexact conserved current, and the positive spectral matrix are important consistency structures. By contrast, multi-field conformal geometry, the general possibility of 2^N vacuum landscapes, conserved vector currents, and Källén–Lehmann positivity are standard ingredients and are not claimed as new.

The broader L(6+7) corpus explored how the same program might be connected to cosmology, photoproduction, temporal order, admissible histories, observer-dependent realization, gravitational response, and other branches [1–17]. The role of the present paper is more limited and more precise: it extracts the local seven-field EFT that is common to those developments and states exactly what can be derived from it without appealing to the phenomenological success of the companion papers.

The appropriate conclusion is therefore that the model provides a structurally complete candidate for a unifying low-energy EFT within the assumptions stated above. It is not an experimentally established theory of everything, it does not select a unique UV completion, and it does not by itself establish that the reference parameter point describes our Universe. Those questions belong to independent matching and empirical tests carried out with the model choices fixed before the relevant data are examined. In particular, the dimensionless tree zero should not be assigned to a named experimental resonance until that matching has fixed the physical energy scale independently of the resonance being tested.

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DATA AVAILABILITY STATEMENT

No experimental or observational data set is analyzed in this paper. All numerical values reported here follow from the analytic equations and parameter values stated in the text. A machine-readable reproduction script and checksum manifest accompany the submission and reproduce the principal numerical quantities and the tree-level P-wave zero. These materials are available from the author upon reasonable request. Any future comparison with independent data will use parameter, matching, and state choices specified before those validation data are examined.

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