Gauge groups, fields and charges from a Hopf bundle with braiding

C.Y. Hsieh
Independent Researcher

19 July 2026

Abstract

We present a purely kinematic construction in which the gauge groups, field content, and matter charges of the Standard Model arise together as multiplexed readouts of a single geometric object: the Hopf bundle $S^3 \xrightarrow{S^1} S^2$, carrying a trivalent connectivity with a braiding on its edges and vertices. A handedness choice on the edge braid breaks parity, selecting the left-handed sector the weak factor acts on. The abelian factor is the fibre's phase holonomy, the non-abelian factors the reconnection algebras of the two-strand edge braid and the three-strand vertex braid, the colour count $N_c = 3$ inherited from the trivalent connectivity. The rank-two edge frame splits into a symmetric spin-2 tensor mode and an antisymmetric spin-1 field strength; spin is the tangent direction, its values fixed by framing topology; the homotopy groups π_1, π_2, π_3 supply the matter charges. The construction fixes what the fields are and how they relate, not how they evolve.

Introduction

A standing aim in mathematical physics is to read the Standard Model's field content off a single geometric object, rather than introducing additional structures to accommodate it. This Letter does so in a strictly kinematic setting. We fix the Hopf bundle $S^3 \xrightarrow{S^1} S^2$ [1, 2], carrying a trivalent connectivity with a braiding on its edges and vertices, and show that the gauge groups, field content, and matter charges of the Standard Model emerge as *multiplexed readouts* of that one structure: distinct observables carried by shared, overlapping degrees of freedom, not separately posited fields. Only the connectivity is fixed; position and direction are relational, read off at each interaction.

The construction sits in the quantum-topology lineage that represents matter and interactions as combinatorial-topological data on a graph: framed trivalent graphs and their braided generalisations [3], knotted field solitons in the manner of Faddeev and Niemi [4], and the braided-preon proposal of Bilson-Thompson [5]. What is specific here is the economy of the input: one bundle-plus-braiding carries the entire readout.

What is fixed is structure, not evolution: group structure, field modes, and charges are extracted; equations of motion, couplings, and masses belong to a dynamical completion.

Bundle and phase space

Each site is a point of one shared Hopf bundle $S^3 \xrightarrow{S^1} S^2$ (Euler class 1), not a bundle of its own, and carries a phase space of five degrees of freedom,

$$\mathcal{Q} \;=\; \mathbb{R}^+ \times S^2 \times S^1_{\text{tan}} \times S^1_{\text{Hopf}}, \tag{1}$$

a tension scalar $\rho \in \mathbb{R}^+$, a position on the base S^2, a tangent direction S^1_{tan}, read as the spin axis, and the fibre phase $\psi \in S^1_{\text{Hopf}}$. The base S^2 and this fibre read out locally as the flat $\mathbb{R}^3$ of space: two directions from the base and the third, depth, from the fibre-phase mismatch between configurations

— phase-resonant configurations read as adjacent, phase-orthogonal ones as depth-separated. The two fields that carry the readouts below are two of these degrees of freedom: the tension ρ and the phase ψ.

Connectivity and braiding

The connectivity is a *trivalent* graph — three edges at each site. As pure adjacency it carries no dimension of its own; the emergent $\mathbb{R}^3$ is read from the bundle, not from the graph. It fixes only which site connects to which, once and for all, and this is where the gauge structure lives. No site carries a coordinate, and no global axis is singled out.

On this graph lives a braiding: a two-strand braid (B_2) along each edge and a three-strand braid (B_3) at each vertex, the three vertex strands linking the three incident edges pairwise (fig. 1). A handedness choice on the edge braid breaks parity: picking one of the two B_2 chiralities fixes a reference configuration.

This locked configuration separates the two strands into a taut anchor and a free carrier, and sets the chirality the weak factor acts on. At each vertex the two exchange roles under the fixed handedness, the anchor continuing as the next bond's carrier and the carrier as its anchor, so a strand recovers its own role only after two vertices, a double cover of its role worldline. This handedness is the only discrete choice the construction makes; everything below is a readout of the braided bundle it fixes — gauge group structure, field content, spin, and matter charges, each stated as an identification that introduces no new structure.

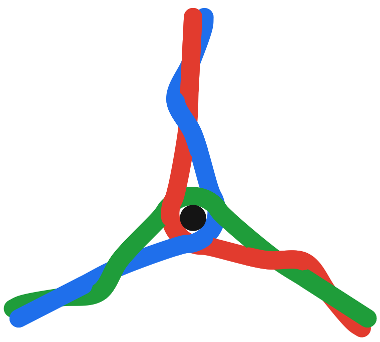

Figure 1: The braiding at a vertex and its three edges, viewed down the axial S^1 fibre (black dot). Each edge carries a two-strand braid B_2; the three strands meeting at the vertex braid as B_3, each linking a distinct pair of the incident edges. The strands depict the topological constraints the braiding places on the connectivity, not physical filaments.

Gauge structure

We read the abelian factor $U(1)$ as the holonomy of the S^1 Hopf-fibre phase — the connection whose curvature is the Euler-class flux on the base. The non-abelian factors we take to be algebras of *strand reconnection*, not the braid groups themselves: the N strands at a braid carry an internal N-plet, and the routings of one strand's content into another span the $N \times N$ matrix algebra $\mathfrak{u}(N) = \mathfrak{su}(N) \oplus \mathfrak{u}(1)$, which the braid renders non-commuting. We accordingly associate the two-strand edge braid B_2 with $\mathfrak{su}(2)_L$, the traceless reconnections of its two strands (the trace settles on the taut anchor, the tension mode below), acting through the fixed handedness on one chirality — the other, carrying no wrap, a singlet — and the three-strand vertex braid B_3 with $\mathfrak{u}(3)$, its eight traceless reconnections the gluons of $\mathfrak{su}(3)_C$ and its trace a conserved bookkeeping charge, baryon number, with no propagating gauge boson attached. The colour count $N_c = 3$ is inherited from the trivalence of the connectivity: three edges meet at a vertex, so three strands braid there. Coupling values belong to the dynamics.

Field content and spin

Since the connection itself is gauge-variant while its curvature is not, we identify the physical electromagnetic field with the curvature two-form $F = dA$, the Euler-class flux on the base. The tensor content follows from squaring the rank-two edge frame (each edge a local direction) and splitting it into symmetric and antisymmetric parts,

$$\text{edge} \otimes \text{edge} \;=\; \text{Sym}^2(\text{edge}) \;\oplus\; \Lambda^2(\text{edge}), \tag{2}$$

the antisymmetric half a spin-1 field strength — one edge direction rotated against the other, representing its three spatial (magnetic) components, with the electric components arriving when the time axis is adjoined — and the symmetric, traceful half a spin-2 tensor mode: the two edge directions stretched together, a change of volume and tension, which we identify as the gravitational tensor mode. The field content read out here is the boson sector — gauge and tensor; matter enters not as a further field but as the topological charges below.

We identify spin with the tangent direction S^1_{tan} — a geometric direction, not an internal rotation — because its framing topology fixes the periodicity of tangent transport. That period sets the value: $360°$ for a single fibre, the spin-1 field strength; $180°$ for the two-leg tensor mode, spin-2; and $720°$ for the anchor–carrier role exchange, spin-$\frac{1}{2}$. Read along any axis, the direction resolves onto an edge, which runs only two ways, so the reading is $\pm$.

Matter charges

The bundle carries one topological charge per homotopy degree, each a readout of a winding no continuous local deformation can undo:

- a *vortex* ($\pi_1(S^1)$), an integer winding of the fibre phase ψ about a core, read as electric charge Q;
- a *skyrmion* [6] ($\pi_2(S^2)$), the strand wound around the axial fibre, read as weak isospin T_3;
- a *hopfion* [4] ($\pi_3(S^2)$), three strands knotted at a vertex into a closed cage of ρ-tension, read as baryon number B.

By the Călugăreanu identity [7, 8, 9] a twist on an edge converts into a writhe at the vertex, but only where it winds something the topology protects; the skyrmion is exactly this conversion, the carrier strand's wrap about the anchor becoming, at the vertex, a wrap about the axial fibre (a writhe on S^2). We identify this twist–writhe exchange as the kinematic form of the weak interaction: what the weak factor converts, in raising or lowering weak isospin, is this wrap.

The vortex and hopfion are stable in three dimensions, their windings unable to unwind, so their charges are conserved. The skyrmion is not: its texture can unwind through the third dimension, so weak isospin, alone among the three charges, is not conserved. Hypercharge adds nothing independent: $Y = 2(Q - T_3)$ fixes it in terms of the electric charge and weak isospin.

The tensor mode admits one further identification. We read a unit of matter as a localised packet of the symmetric tension mode, held against dispersal by the topological charges its region carries; the same mode, free and unpinned, is the propagating tensor field. Matter and gravity are thus the *trapped* and *propagating* manifestations of the same rank-two framed mode. Under this identification, localised trapped excitations naturally act as sources for the propagating mode. This much is kinematic; the form of the coupling, a field equation, is a dynamical question.

Illustrative particle assignments

With the kinematic readouts established, the same braided bundle admits a natural organisation of elementary particles into distinct topological sectors.

Electric charge is the vortex winding of the fibre phase (π_1): on the fibre a holonomy, on the base its Coulomb flux. A charged lepton is a single such vortex, its one winding the unit of electric charge, and it carries no colour. The carrier strand may wrap the vortex core; the number of wraps — zero, one, or two — labels the three generations, the charge unchanged throughout. What caps the count at three

— forbidding a third wrap — is a dynamical saturation outside this construction, so the generation number is taken here as input. The electron is the plain, unwrapped vortex.

A neutrino carries no integer winding. The carrier's residual twist organises into three nested modalities: a twist of the strand about its own axis, a wrap around its anchor, and a collective twist of the whole edge as one Hopf fibre. A neutrino is one of these modalities excited below any integer winding.

A baryon closes colour locally: three strands knot at one vertex into a π_3 hopfion, a closed cage of ρ-tension, and the baryon is that cage, baryon number counting the cages. The knot is an integer class no continuous deformation undoes, hence topologically protected and conserved, so the simplest such cage is stable. A meson closes colour non-locally, a strand bridging to a conjugate through a colour flux tube; tying no protected knot, it is a legal but unprotected concentration.

The knot is itself a second Călugăreanu conversion, now of the whole edge: its twist as one Hopf fibre becomes its linked writhe at the vertex. Its chirality is therefore fixed by the construction's one handedness choice, and on this ground we conjecture that CP breaking and baryogenesis are structurally related.

The gauge bosons are the complement of these matter packets: not stored charges but the propagating disturbances the reconnection operators leave in the two fields. The photon stays within the phase field ψ and the gluon within the tension field ρ, while the electroweak W and Z bridge the two. The $SU(2)_L$ conversion adds or removes a wrap, moving it between its twist form on the edge and its writhe form at the vertex; its charged form $W^{\pm}$ is what turns a down-type into an up-type. The skyrmion texture can unwind, so the conversion violates no conserved charge.

A quark is one strand's structure inside a vertex, its charge fixed by three-fold closure. Let each strand carry charge q; the trivalent vertex sums its three strands to $3q$, and the weak conversion shifts a single strand by one unit — $W^{\pm}$ adds or removes a wrap — giving vertex totals $3q,\, 3q \pm 1,\, \ldots$. That total, the integer charge of the colourless composite, forces $3q \in \mathbb{Z}$, so q is a multiple of $\frac{1}{3}$; the minimal non-trivial value $q = -\frac{1}{3}$ gives, under one weak shift, $+\frac{2}{3}$ — the down- and up-type charges. No strand has a free state: the tension holds only the colour-closed vertex, so confinement here is a structural identity, not a dynamical trap.

Scope and outlook

The construction invokes no length unit: it rests on topology alone — bundle, homotopy, braiding, connectivity. With no spacing, it is relational rather than lattice-like, and the checkerboard artefacts of fixed-lattice discretisations have nothing to attach to.

The gravitational sector shows the kinematic boundary sharply: we identify the tensor mode and name matter and gravity its trapped and propagating manifestations, but a field equation would need a time axis the construction withholds, so we stop at the identification. A completion supplying that axis would still find some of the equation already prefigured. Were the symmetric and antisymmetric sectors promoted to tension and phase dynamics on the bundle's total space, the flux normalisation would produce the familiar coefficients — 4π for a point source over the S^2 solid angle, doubled to 8π by the symmetric mode's trace reversal, bare for the traceless field strength; a geometric consistency check for such a completion, not a derivation. The locked configuration's baseline tension would then enter as a cosmological term, not a divergent zero-point energy, matter gravitating only through its excess; and mass would come from the tension field's amplitude mode, the Higgs, whose elastic dynamics about the vacuum sets the trapped stress of each packet.

Acknowledgements

The author thanks Anthropic for providing AI writing tools (Claude Fable 5), used for correcting English grammar and wording, improving fluency, and editing LaTeX. The author reviewed and edited the output as needed and takes full responsibility for the content of the published article.

References

[1] H. Hopf, *Math. Ann.* **104** (1931) 637.

[2] H. K. Urbantke, *J. Geom. Phys.* **46** (2003) 125.

[3] V. G. Turaev, *Quantum Invariants of Knots and 3-Manifolds* (de Gruyter, Berlin, 1994).

[4] L. Faddeev and A. J. Niemi, *Nature* **387** (1997) 58.

[5] S. O. Bilson-Thompson, *A topological model of composite preons*, arXiv:hep-ph/0503213 (2005).

[6] T. H. R. Skyrme, *Nucl. Phys.* **31** (1962) 556.

[7] G. Călugăreanu, *Czech. Math. J.* **11** (1961) 588.

[8] J. H. White, *Am. J. Math.* **91** (1969) 693.

[9] F. B. Fuller, *Proc. Natl. Acad. Sci. USA* **68** (1971) 815.