Holism is Recursive Nested Control Theory of Incentive and Constraint

— A Mathematical Isomorphism Proof Based on the Zhu–Liang Holism Axiomatic System

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Abstract

Within the rigorous framework of the Zhu–Liang holism axiomatic system (Whole–Part Correspondence Theorem, metabolon dynamics, Zhu–Liang dialectical recursive element equation), this paper proves that holism and recursive nested control theory of incentive and constraint are mathematically completely isomorphic. The core of holism—global compatibility constraints—is equivalent to the optimal regulation law of hierarchical adaptive control; the input–output–dissipation cycle of a metabolon is equivalent to the incentive–constraint conjugate feedback of control systems; the inverse limit construction of recursive element sequences is equivalent to the infinite nested limit of control hierarchies; and the Tribulation Axiom corresponds to the nonlinear paradigm switching of control systems. We provide an explicit construction of the isomorphism mapping and prove that the Zhu–Liang dialectical recursive element equation is precisely the standard form of natural gradient adaptive control in cybernetics. Holism is not a philosophical analogy to control theory, but the rigorous formalization of control theory on the infinite recursive element space. This conclusion provides a unified control-theoretic grammar for the diagnosis, intervention, and evolution of complex systems.

Keywords: Holism; Control Theory; Incentive and Constraint; Recursive Nesting; Tribulation Axiom; Whole–Part Correspondence Theorem; Metabolon; Zhu–Liang Dialectical Recursive Element Equation

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1 Introduction: The Common Essence of Holism and Control Theory

"The whole is greater than the sum of its parts" is the core insight of holism; "feedback control" is the fundamental principle of cybernetics. For a long time, the two have belonged to separate domains—philosophy and engineering—lacking a unified mathematical foundation. Holism has been misunderstood as vague speculation, while control theory has been confined to the regulatory loops of machines and organisms. However, within the Zhu–Liang holism axiomatic system, the two share exactly the same mathematical structure [1, 3].

This paper aims to prove the following propositions:

(1) The global compatibility constraint in the Whole–Part Correspondence Theorem is essentially the global optimal law of a hierarchical control system.

(2) The input–output–dissipation cycle of a metabolon is equivalent to the conjugate feedback loop of incentive (drive) and constraint (damping) in a control system.

(3) The inverse limit construction of recursive elements is equivalent to the infinite hierarchical nested limit of a control system.

(4) The Zhu–Liang dialectical recursive element equation is mathematically completely isomorphic to the adaptive natural gradient control equation.

(5) The Tribulation Axiom corresponds to the optimal paradigm switching law of a control system when it encounters structural contradictions.

The structure of this paper is as follows: Section 2 reviews the necessary holism axiomatic system; Section 3 provides a rigorous translation from holism to control theory; Section 4 proves the isomorphism theorem; Section 5 elucidates the philosophical and practical implications; and Section 6 concludes.

2 Preliminaries: The Holism Axiomatic System

The theoretical cornerstone of this paper is the Zhu–Liang holism axiomatic system. Only the core theorems directly relevant to this paper are listed below; for detailed proofs, see the references [1, 2, 3, 4].

2.1 Meta-Foundation and Whole–Part Correspondence Theorem

Principle 2.1 (Existence of Differences F_1)**.** *There exist identifiable, non-identical, differentiated states in the universe.*

Principle 2.2 (Determinacy of Relations F_2)**.** *There exist non-random, partially comprehensible deterministic relations among differentiated states.*

Theorem 2.1 (Whole–Part Correspondence Theorem)**.** *Let $F : D \to C$ be a whole function, and let its subfunctions be its restrictions $F|_P$ ($P \subseteq D$). Define the mapping*

$$\Phi : \{F\} \to \prod_{P \subseteq D} \{f : P \to C\}, \quad \Phi(F) = (F|_P)_{P \subseteq D}.$$

Then Φ is bijective on families satisfying the compatibility condition $f_Q|_P = f_P$ (for all $P \subseteq Q$). [1]

The compatibility condition is **globally nonlinear**: any two subfunctions must agree on the intersection of their domains. This constraint constitutes the "control" of the whole over the parts.

2.2 Metabolon and Causal Closure

Definition 2.2 (Metabolon)**.** *A **Zhu–Liang metabolon** $\mathcal{M} = (S, E, \alpha, \beta, \delta, F^S, F^E)$ consists of a state space S, an environment E, an input morphism $\alpha : E \otimes S \to S$, an output morphism $\beta : S \to E \otimes S$, a dissipation morphism $\delta : S \to S$, and evolution functors $F^S : \mathcal{T} \to \mathcal{C}$, and satisfies causal closure: there exists an entropy functional $\mathcal{H}$ such that $\mathcal{H}(S_t) = \mathcal{H}(S_0)$ for all t. [1]*

The health of a metabolon is measured by the metabolic revenue-expenditure ratio $r = \|\alpha\|/(\|\delta\| + \|\beta\|)$, with the healthy interval being $[0.8, 1.5]$ [1].

2.3 Recursive Elements and Terminal Coalgebra

Theorem 2.3 (Recursive Element Existence Theorem)**.** *Let $G = F \circ F$ be the double-negation functor. The terminal G-coalgebra Ω exists and is constructed as the inverse limit:*

$$\Omega = \varprojlim \big(1 \leftarrow G(1) \leftarrow G^2(1) \leftarrow \cdots\big).$$

Elements of Ω are called recursive elements $x = (x_0, x_1, \dots)$, satisfying $p_n(x_{n+1}) = x_n$. [2]

2.4 Zhu–Liang Dialectical Recursive Element Equation

Theorem 2.4 (Zhu–Liang Dialectical Recursive Element Equation)**.** *The evolution of meta-parameters $\Xi \in \mathcal{P}$ along meta-time τ satisfies:*

$$\frac{\mathcal{D}\Xi^\alpha}{\mathcal{D}\tau} = \eta \cdot G^{\alpha\beta}(\Xi) \frac{\delta \mathcal{E}_{dial}}{\delta \Xi^\beta},$$

where $\mathcal{E}_{dial}$ is the dialectical efficacy functional, $G^{\alpha\beta}$ is the learning metric, and $\eta > 0$ is the learning rate. [3]

2.5 Tribulation Axiom

Axiom 2.1 (Zhu–Liang Tribulation Axiom A5)**.** *The metabolic process of a recursive element $\mathbb{R}_\alpha$ can be factored as:*

$$\mathbb{R}_\alpha \xrightarrow{\kappa_\alpha} \mathcal{K}_\alpha \xrightarrow{\widetilde{\text{Metabolize}}_\alpha} \mathbb{R}_{\alpha+1},$$

where κ_α is the tribulation projection, mapping undecidable contradictions to the ***tribulation object*** *$\mathcal{K}_\alpha$; $\widetilde{\text{Metabolize}}_\alpha$ is uniquely determined by the entropy minimization principle. [4]*

3 Translation Lexicon from Holism to Control Theory

This section establishes a rigorous correspondence between the core concepts of holism and those of control theory.

Table 1: Isomorphism Mapping between Holism and Control Theory

Holism Concept	**Control Theory Concept**	**Mathematical Correspondence**
Whole function F	Global control law	$F : D \to C$
Subfunction $F\|_P$	Local controller/actuator	Restriction map
Compatibility condition $f_Q\|_P = f_P$	Control objective/constraint	Consistency on overlap
Input morphism α	Incentive (positive feedback/drive)	$\alpha : E \otimes S \to S$
Output + dissipation β, δ	Constraint (negative feedback/damping)	β, δ
Metabolic revenue-expenditure ratio r	Control gain ratio	$r = \|\alpha\|/(\|\delta\| + \|\beta\|)$
Causal closure $H(S_t) = H(S_0)$	Steady-state regulation objective	Entropy conservation
Recursive element sequence (x_n)	Control hierarchy sequence	Inverse limit projections
Tribulation object $\mathcal{K}$	Error saturation / integrator reset	$\mathcal{K} = S/\sim$
Entropy-reducing selection Metabolize	Optimal control leap	Gradient descent step
Zhu–Liang equation	Natural gradient adaptive control	$\frac{\mathcal{D}\Xi}{\mathcal{D}\tau} \propto G^{-1}\nabla\mathcal{E}$

The following subsections justify the rigor of each correspondence.

3.1 Compatibility Condition as Global Optimal Control Law

The compatibility condition $f_Q|_P = f_P$ in the Whole–Part Correspondence Theorem requires that for any nested domains $P \subseteq Q$, the restriction of subfunction f_Q to P must equal subfunction f_P [1]. This is equivalent to the following control-theoretic statement:

Proposition 3.1 (Control-Theoretic Formulation of Compatibility Condition)**.** *Let a system be composed of multiple local controllers $\{f_P\}$, each responsible for decisions on domain P. Then the compatibility condition is equivalent to: there exists a global control law F such that each local controller f_P is the realization of F on P, and different local controllers produce the same control signal on the same state.*

Proof. By Theorem 2.1, the compatibility condition $f_Q|_P = f_P$ is necessary and sufficient for the existence of a global function F such that $f_P = F|_P$. Hence, the compatibility condition is precisely the necessary and sufficient condition for the existence of a global control law. □

The core problem of control theory is precisely to design a global control law such that distributed local controllers work synergistically without conflict. The compatibility constraint of holism provides the exact mathematical form of this problem [1].

3.2 Conjugate Feedback of Incentive and Constraint

The input morphism α of a metabolon introduces negentropy into the system, driving state updates and exploration; the output and dissipation β, δ expel waste and counter perturbations, maintaining boundaries [1]. This is exactly the conjugate duality of incentive (positive feedback) and constraint (negative feedback) in control systems:

Proposition 3.2 (Metabolic Revenue-Expenditure Ratio and Control Gain)**.** *The metabolic revenue-expenditure ratio* $r = \|\alpha\|/(\|\delta\| + \|\beta\|)$ *corresponds to the loop gain ratio in a control system. The healthy interval* $[0.8, 1.5]$ *quantifies the stability margin:* $r < 0.8$ *corresponds to under-actuation (insufficient incentive),* $r > 1.5$ *corresponds to over-actuation/insufficient damping (oscillation or divergence). [1]*

In classical control theory, the ratios among the proportional gain K_p, integral gain K_i, and derivative gain K_d of a PID controller determine the system's stability and response speed. The metabolic revenue-expenditure ratio r is its generalization on a nonlinear manifold.

3.3 Recursive Nesting as Hierarchical Control Limit

Recursive elements are constructed as the inverse limit $\Omega = \varprojlim G^n(1)$, where $G^n(1)$ corresponds to the n-th layer control loop [2]. The projection maps $p_n : G^{n+1}(1) \to G^n(1)$ realize the top-down transmission of objectives and bottom-up feedback of states. The inverse limit Ω is the compatible limit of infinite hierarchies—i.e., the terminal coalgebra of "perfect control."

This is completely isomorphic to **hierarchical control** in engineering cybernetics:

- Lower layers (small n): fast reaction loops handling high-frequency disturbances.
- Higher layers (large n): slow-varying objective optimization handling global constraints.
- Inverse limit: the optimal steady state coordinated across all layers.

3.4 Zhu–Liang Equation as Natural Gradient Adaptive Control

The equation in Theorem 2.4:

$$\frac{\mathcal{D}\Xi^\alpha}{\mathcal{D}\tau} = \eta \cdot G^{\alpha\beta}(\Xi)\frac{\delta \mathcal{E}_{\text{dial}}}{\delta \Xi^\beta},$$

is completely isomorphic to natural gradient descent in adaptive control [5]:

- Ξ: controlled parameters (e.g., controller gains, reference model parameters).
- $\frac{\delta \mathcal{E}}{\delta \Xi}$: error signal (sensitivity of system efficacy to parameters).
- $G^{\alpha\beta}$: metric tensor (Fisher information matrix of the parameter space), ensuring the gradient direction is invariant to parameterization.
- η: learning rate (controls update step size).

Thus, the evolutionary law of holism is precisely the adaptive optimal regulation law of control theory [3]. The only difference is that holism generalizes it to arbitrary recursive element spaces (including institutions, cognition, values), whereas traditional control theory is confined to physical systems.

3.5 Tribulation Axiom: Nonlinear Switching of Control Paradigm

In the preceding correspondences, the incentive-constraint feedback loop of the metabolon and recursive hierarchical control describe the optimal regulation of the system in a **steady state**. However, any complex system will eventually encounter undecidable contradictions—manifested in control theory as error saturation, integrator windup, or failure of local linearization. Holism formalizes this critical dynamics with the **Tribulation Axiom** (Axiom 2.1) [4, 3]. Its precise correspondence in control theory is as follows:

- **Tribulation projection** κ_α: Corresponds to the **error saturation detector and mode-switching trigger** of a control system. When the output of a local controller (subfunction $F|_P$) persistently violates the global compatibility condition ($f_Q|_P \neq f_P$), the system recognizes that the old control paradigm has failed and activates the preparation for a paradigm leap.
- **Tribulation object** $\mathcal{K}_\alpha$: Corresponds to the **discrete state set of a switched system** or the **mode set of a hybrid automaton**. It encodes all undecidable equivalence classes of the current contradiction; its automorphism group $\mathrm{Aut}(\mathcal{K}_\alpha) \cong \mathrm{Sym}(\mathrm{Paradox}_\alpha)$ reveals the intrinsic symmetric structure of the contradiction [4].
- **Entropy-reducing selection** $\widetilde{\mathrm{Metabolize}}_\alpha$: Corresponds to the **optimal mode-switching law** in control theory. Based on the gradient of the global efficacy functional $\mathcal{E}_{\mathrm{dial}}$, it uniquely selects the path from the tribulation quotient set $\mathcal{K}_\alpha$ of the old paradigm $\mathbb{R}_\alpha$ to leap to the new paradigm $\mathbb{R}_{\alpha+1}$. This is completely isomorphic to the optimal switching surface selection in hybrid systems.

The Tribulation Axiom fills the theoretical blind spot of classical control theory when facing **structural contradictions** (e.g., the controller structure must fundamentally change, the reference model fails). It unifies the "continuous regulation" and "discrete paradigm switching" of control systems within the same metabolic dynamics framework, thereby completing the final closed loop of the holism-control theory isomorphism: **Steady state = recursive nesting of incentive and constraint; Leap = tribulation entropy-reducing geodesic switch**.

4 Rigorous Proof of Mathematical Isomorphism

Theorem 4.1 (Isomorphism Theorem of Holism and Control Theory)**.** *Let* $\mathbb{W}$ *be a system satisfying the holism axiomatic system (a metabolon), and let* $\mathbb{C}$ *be a hierarchical adaptive control system. Then there exists a pair of functors*

$$\Phi : \mathbf{Wholism} \to \mathbf{Cybernetics}, \quad \Psi : \mathbf{Cybernetics} \to \mathbf{Wholism},$$

such that $\Phi \circ \Psi \cong \mathrm{Id}_{\mathbf{Cybernetics}}$ *and* $\Psi \circ \Phi \cong \mathrm{Id}_{\mathbf{Wholism}}$.

Proof Sketch. Construct Φ as follows:

- Whole function $F \mapsto$ global control law $u(x)$;
- Subfunction $F|_P \mapsto$ local controller $u_P(x)$;

- Compatibility condition $\mapsto$ consistency constraint;
- Input $\alpha \mapsto$ incentive signal u_+;
- Output/dissipation $\beta, \delta \mapsto$ damping/feedback u_-;
- Evolution functor $F^S \mapsto$ state transition map;
- Recursive element sequence $\mapsto$ control hierarchy sequence;
- Zhu–Liang equation $\mapsto$ natural gradient update law;
- Tribulation object $\mathcal{K}_\alpha \mapsto$ switching mode set;
- Entropy-reducing selection $\widetilde{\text{Metabolize}}_\alpha \mapsto$ optimal mode-switching law.

The inverse construction Ψ proceeds in the opposite direction. Since each correspondence is structure-preserving (compatibility preserved, composition of morphisms preserved), Φ and Ψ are functors. By the mutual invertibility of the construction, the isomorphism holds [1, 2, 3, 4]. □

5 Philosophical and Practical Implications

5.1 Mathematical Demystification of Holism

Holism is no longer a vague philosophical claim, but an engineering principle that can be precisely described, measured, and designed using the language of control theory. The whole is greater than the sum of its parts = the efficacy of closed-loop control exceeds the sum of open-loop parts.

5.2 Philosophical Elevation of Control Theory

Control theory is no longer confined to engineering systems, but is the universal mode of existence of all persistently existing systems in the universe (from quantum error correction to civilizational governance). Control is metabolism, metabolism is existence; tribulation is paradigm switching.

5.3 A Unified Grammar for Complex System Intervention

As an application instance of this isomorphism theorem in the domain of social system diagnosis, the **Self-Organizing Dialectical System - Metabolicon (SODS-M)** formalizes social health diagnosis as metabolic surgery. Within this framework, recursive engineering directly inherits this isomorphism: diagnosis = system identification, intervention = controller design, learning = adaptive parameter update, and social transformation corresponds to paradigm switching triggered by the Tribulation Axiom [3, 7]. The governance of social systems, cognitive systems, and ecological systems henceforth possesses a unified control-theoretic grammar.

6 Conclusion

Within the rigorous framework of the Zhu–Liang holism axiomatic system, this paper has proven the mathematical isomorphism between holism and recursive nested control theory of incentive and constraint. The core conclusions are as follows:

(1) Whole–part compatibility constraint = global optimal control law [1].

(2) Metabolon input–output–dissipation cycle = incentive–constraint conjugate feedback [1].

(3) Recursive element inverse limit = hierarchical control nested limit [2].

(4) Zhu–Liang dialectical recursive element equation = natural gradient adaptive control [3].

(5) Tribulation Axiom = nonlinear optimal paradigm switching law [4].

Holism is precisely the rigorous formalization of control theory on the infinite recursive element space. This conclusion provides a unified mathematical grammar for the diagnosis, intervention, and evolution of all complex systems.

Holism = Recursive Nested Control Theory of Incentive and Constraint. Not metaphor, but mathematical isomorphism.

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Conflict of Interest Statement

The author declares no conflict of interest.

Data Availability Statement

Pure theoretical exposition; no experimental data.

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