

Without Rigorous Logic, How Can There Be Science and Law?

— The Ultimate Confirmation of the Foundations of Rational Civilization

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DOI: [10.5281/zenodo.19478029](https://doi.org/10.5281/zenodo.19478029)

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Preprint submitted: April 9, 2026

Abstract

Science and law are two foundational normative systems of human civilization. Science regulates our beliefs about the natural world, while law regulates our actions in the social world. This paper demonstrates that both share the same necessary condition in their structure: **rigorous logic**. Logic is not an ex post addition but the **transcendental grammar** and **first condition of existence** for science and law. The process by which science extracts universal laws from discrete experiences is a recursive nesting of logical entropy reduction; the process by which law condenses generalizable rules from conflicts of multiple values is likewise a tribulation-transcending leap of logical structure from high entropy to low entropy. Logical rupture inevitably leads to pseudoscience and the absence of the rule of law. Based on the Zhu-Liang theoretical chain (MPD[4], SODS[5], Holism Theorem[1], Truth Degree Determination Theorem[2], Recursive Nesting Theorem[3]), this paper proves at the meta-theoretical level that the validity of science and law entirely depends on the rigor of their internal logic; without logic, neither can exist conceptually. This proposition is an analytic truth whose self-evidence constitutes the unshakable foundation of rational civilization.

Keywords: logic; philosophy of science; jurisprudence; transcending tribulations; entropy reduction; recursive unit; normative systems

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1 Introduction: Logic as the Metalanguage of Rational Civilization

"Without rigorous logic, how can there be science and law?" This seemingly simple question directly strikes at the core of rational civilization. It is not an empirical question (whether there have ever been sciences or laws without logic in history) but a conceptual one: does the definition of science and law already entail the necessity of logic?

The answer of this paper is affirmative. Science uses logic as its grammar to organize observational facts into a deducible, predictable theoretical system; law uses logic as its skeleton to arrange social values into an applicable, adjudicable normative order. If logic is removed, science degenerates into a pile of fragmented experiences, and law degenerates into an arbitrary declaration of power both lose their qualification as achievements of civilization.

This paper systematically argues for this proposition from five levels: logic as transcendental grammar, the logical structure of science, the logical structure of law, the consequences of logical rupture, and the tribulation-transcending and entropy-reduction mechanisms in the Zhu-Liang theoretical chain. Ultimately, it proves that logic is not outside science and law but within them; not after them but before them.

2 Logic: The Transcendental Grammar of Science and Law

2.1 The Concept of Transcendental Grammar

"Transcendental grammar" refers to the set of logical rules that any normative system called a "scientific theory" or "legal system" must follow a priori. These rules are not learned conventions but the conditions of possibility for rational discourse. Just as Kant in the *Critique of Pure Reason*[6] argued that space, time, and categories are the transcendental forms of experience, this paper maintains that logic is the transcendental form of science and law.

Definition 2.1 (Transcendental Grammar). *Let \mathcal{S} be a normative system (scientific theory or legal system). The **transcendental grammar** of \mathcal{S} is a set of rules \mathcal{L} satisfying:*

1. *If \mathcal{S} does not obey \mathcal{L} , then \mathcal{S} cannot maintain its identity as a normative system;*
2. *\mathcal{L} cannot be derived from the empirical content of \mathcal{S} but is the precondition for the possibility of \mathcal{S} .*

2.2 Core Logical Rules

The core logical rules shared by science and law include:

1. **Law of Identity:** $A = A$. In science, concepts must retain the same meaning; in law, terms must be interpreted consistently.
2. **Law of Non-Contradiction:** $\neg(A \wedge \neg A)$. A scientific theory cannot simultaneously assert a proposition and its negation; legal norms cannot simultaneously require an action and prohibit it.

3. **Law of Excluded Middle:** $A \vee \neg A$. A scientific proposition is either true or false; a legal dispute is either upheld or dismissed (no "third answer").
4. **Law of Causal Implication Transitivity:** $(A \rightarrow B) \wedge (B \rightarrow C) \rightarrow (A \rightarrow C)$. The chain of scientific deduction must be closed; the chain of legal reasoning from major premise to minor premise to conclusion must be coherent.

Among these, the law of causal implication transitivity is the most fundamental, as it constitutes the **necessary basis** for scientific prediction and legal adjudication. Without transitivity, from "all metals conduct heat when electrified" and "copper is a metal" one cannot deduce "copper conducts heat when electrified" scientific experiments become irreproducible and legal precedents become inapplicable.

2.3 The Self-Referential Necessity of Logic

Any attempt to deny logic falls into a self-referential paradox: the assertion "logic is useless" itself uses logical inference (from "logic is useless" to "one should not use logic" requires logical reasoning). Therefore, logic is transcendental it is not an optional tool but the inescapable ground of rational discussion.

Proposition 2.2 (Self-Referential Necessity of Logic). *For any rational discourse \mathcal{D} that intends to assert some proposition P , \mathcal{D} must presuppose the logical rules \mathcal{L} . Even if \mathcal{D} asserts "logic is invalid," that assertion itself depends on logical rules (e.g., the law of excluded middle: either logic is valid or logic is invalid). Hence, logic is an **undeniable presupposition** of rational discourse.*

2.4 Equality Before Logic

"Equality before logic" is an absolute truth confirmed by this framework.

Logic is the transcendental grammar of rational discourse. Any proposition claiming truth value, any argument attempting to reach understanding, any construction of a normative system must obey without exception the laws of identity, non-contradiction, excluded middle, and causal implication transitivity. This set of rules does not change with the subject's identity, status, wealth, belief, or history the king and the peasant are equally bound in logical reasoning; the judge and the defendant occupy symmetric positions in logical argumentation.

If inequality existed before logic, it would mean that some class of subjects is permitted to violate the law of non-contradiction (e.g., simultaneously asserting A and non- A) or is exempt from the law of causal transitivity (e.g., from "all A are B " and " x is A " cannot deduce " x is B "). This would lead to the collapse of rational discourse: any conclusion could be arbitrarily declared, and the boundary between truth and falsehood would disappear. Therefore, logical equality is not a product of social contract but an internal requirement for the possibility of rationality.

In science and law, this equality manifests as: the scientific community applies the same deductive rules to everyone; the deep foundation of "equality before the law" in legal systems is precisely equality before logic because if legal reasoning applied different logical rules to different people, the rule of law would cease to exist. Hence, "equality before logic" is a more fundamental principle than "equality before the law," constituting the unshakable cornerstone of rational civilization.

3 Science: From Empirical Fragments to Logical Closure

3.1 Observational Facts and Theoretical Logic

The starting point of scientific research is observational facts—discrete, sometimes contradictory perceptual data. Observational facts alone do not constitute science. Astronomers in Ptolemy's time observed the same apparent motions of the planets but constructed the cumbersome system of epicycles and deferents; Copernicus and Kepler, facing the same facts, extracted elliptical orbits and heliocentrism. The difference lies in the **logical structure**.

Example 3.1 (Ptolemaic System vs. Keplerian System). *The Ptolemaic system set independent epicycle parameters for each planet, lacking a unified causal principle. The Keplerian system used a single principle (the sun's gravity decreasing with distance) to explain all planetary motions. The former is logically redundant (high entropy), the latter logically concise (low entropy). Scientific progress is precisely the transition from high-entropy to low-entropy logical structures.*

3.2 Hypothetico-Deductive Method as a Logical Closure Procedure

The core of modern scientific methodology—the hypothetico-deductive method—is essentially a logical closure procedure. Popper in *The Logic of Scientific Discovery*[7] emphasized that the falsifiability of scientific theories depends on logical derivation relations. The steps are:

1. Propose axioms or basic hypotheses H ;
2. Derive testable consequences P through logical deduction $H \vdash P$;
3. Verify P experimentally;
4. If P is false, revise H (the logical premises).

Without logic, step 2 cannot be executed. Science would degenerate into primitive "observation-guess" magic. Therefore, science becomes science precisely because it incorporates experience into a **recursively derivable logical network**—this is the epistemological manifestation of the "Recursive Nesting Theorem"[3] in the Zhu-Liang theoretical chain.

Theorem 3.2 (Logical Necessity of Science). *Let \mathcal{T} be a scientific theory and \mathcal{O} its corresponding set of observational facts. If there is no logical derivation relation \vdash from axioms to observational propositions within \mathcal{T} , then \mathcal{T} can neither be falsified nor generate new predictions. Hence, the logical derivation relation is a necessary condition for a scientific theory to be a testable and progressive knowledge system.*

3.3 Scientific Revolutions as Logical Tribulation-Transcending

In the Zhu-Liang theoretical chain, tribulation-transcending is defined as the process by which a system achieves a leap by identifying contradictions (tribulation objects) and performing entropy-reducing selection (logical reconstruction)[2]. Scientific revolutions are paradigmatic examples of this process:

- **Tribulation projection:** In the face of anomalies such as the Michelson-Morley experiment, classical physics revealed logical fissures (the contradiction between the constancy of the speed of light and Galilean transformations).
- **Entropy-reducing selection:** Einstein abandoned absolute spacetime (redefining the equivalence relation \sim) and adopted Lorentz transformations, leaping to the relativistic framework.
- **New steady state:** Relativity is logically more coherent, has stronger predictive power, and has lower entropy.

Without logic, there would be no criterion for identifying fissures and no criterion for selecting new frameworks—scientific revolutions would degenerate into blind mutations.

4 Law: From Value Conflicts to Normative Order

4.1 The Logical Pyramid of Law

The starting point of law is conflicts among multiple values. Freedom and security, efficiency and fairness, individual rights and the public interest—these values themselves cannot be ranked by logic. However, what distinguishes a legal system from simple rule by violence is that it incorporates these conflicts into a **logically self-consistent normative hierarchy**. As Hart stated in *The Concept of Law*[8], the core of a legal system lies in the logical combination of rules of recognition and secondary rules.

Definition 4.1 (Normative Hierarchy). *A legal system \mathcal{L} is a directed graph of norms where:*

- *The constitution C is at the top;*
- *Lower-level norms L_i must satisfy $C \vdash L_i$ (i.e., L_i is a logical consequence of C or within its authorization);*
- *A judgment J must satisfy $L_i \wedge F \vdash J$, where F are the facts.*

The vitality of this pyramidal normative system depends entirely on the transitivity of logic: if a higher-level norm prescribes A and a lower-level norm prescribes B , and B conflicts with A , then B is invalid. Without this logical relation, law would be a mere collection of isolated commands, judges could arbitrarily choose which to apply, and justice would become accidental.

4.2 The Logical Order of Legal Interpretation

The order "literal interpretation systematic interpretation historical interpretation teleological interpretation" in legal hermeneutics is essentially a **priority ordering of logic**:

- Literal interpretation determines the literal logical boundary (law of identity);
- Systematic interpretation ensures absence of contradiction among norms (law of non-contradiction);
- Historical interpretation traces the logical consistency of legislative intent;
- Teleological interpretation ultimately unifies under the rational purpose of law (law of causal transitivity).

Each step depends on logical operations. If the logical order is abandoned, interpreters can arbitrarily choose any interpretive method to reach any conclusion legal certainty vanishes.

4.3 The Logical Essence of the Rule of Law

The core difference between the rule of law and rule by law is that the rule of law requires law itself to possess internal logical consistency, and government actions must be derivable from the law by logical deduction. In the language of the Zhu-Liang framework: the rule of law requires the legal system to be a **recursive unit** from constitutional axioms, specific judgments are uniquely determined through logical derivation. Holmes in *The Common Law*[9] emphasized that "the life of the law has not been logic: it has been experience," but experience can only become norm through logic.

Theorem 4.2 (Logical Conditions for the Rule of Law). *A society is under the rule of law if and only if its legal system \mathcal{L} satisfies:*

1. *There exists a finite set of axioms (constitutional principles);*
2. *All lower-level norms and judgments are logical consequences of these axioms;*
3. *The derivation relation \vdash is deterministic (given the same facts, different judges independently reach the same conclusion).*

If logical derivation is non-unique or non-existent, the rule of law degenerates into the will to power.

4.4 Legal Evolution as Tribulation-Transcending

When a legal system exhibits logical contradictions for example, when a new law conflicts with an old law without an explicit repeal clause, or when different courts reach opposite judgments on the same facts the rule of law falls into crisis. At such times, "tribulation-transcending" is required:

- **Tribulation object:** Logical fissures condense into undecidable normative conflicts (e.g., the issue of slavery left unresolved in the 1787 U.S. Constitution).

- **Entropy-reducing selection:** Through legislative amendments, judicial interpretation, or constitutional adjudication, logical fissures are eliminated and the normative system is restored to a low-entropy state.
- **New steady state:** For example, the Thirteenth, Fourteenth, and Fifteenth Amendments to the U.S. Constitution, which carried the logical premise "all men are created equal" to its conclusion.

Thus, Holmes's dictum that "the life of the law has not been logic: it has been experience" is one-sided: **the integration of experience must ascend to law through logic.**

5 Degeneration at the Point of Logical Rupture: Pseudoscience and the Absence of the Rule of Law

5.1 Logical Diagnosis of Pseudoscience

Pseudoscience cannot progress precisely because its logical structure has an **irreparable rupture**.

Example 5.1 (Logical Rupture of Astrology). *Astrology claims that celestial positions influence human fate but provides no causal mechanism and cannot construct falsifiable consequences. Its logical chain breaks at the link "celestial positions character/fate" without the law of transitivity, from "Mars is in such-and-such constellation" to "you will have bad luck today" there is no necessary derivation.*

Example 5.2 (Logical Rupture of Alchemy). *Alchemy could not establish a conservation rule for elemental transformation (the law of mass conservation had not yet been discovered), so it remained forever in a trial-and-error stage, unable to form a derivable theoretical system.*

In the language of the tribulation-transcending framework: the tribulation object \mathcal{K} of pseudoscience is pathological either \mathbf{P} is empty (no contradiction, thus no evolutionary driving force), or the equivalence relation \sim is not well-defined (contradictions cannot condense), or the entropy-reducing selection $\widetilde{\text{Metabolize}}$ is not unique (multiple mutually exclusive "new theories" cannot be ruled out). Hence, pseudoscience cannot complete tribulation-transcending, and the knowledge system stagnates in entropy increase.

5.2 Logical Diagnosis of the Absence of the Rule of Law

When a legal system is arbitrarily distorted by the will to power, when judgments are given without reasons, when different judgments on the same case become the norm, law degenerates into an instrument of rule. At that point, logic is expelled and the rule of law perishes.

Example 5.3 (Legal Degeneration in Nazi Germany). *In Nazi Germany, law still existed formally, but logical consistency was replaced by the "Führer principle" any legal provision conflicting with the Führer's will was deemed invalid. The law of transitivity was destroyed, and law degenerated into arbitrary proclamation.*

Therefore, the fundamental criterion for measuring whether a society is under the rule of law is not the number of legal provisions but the **logical necessity** in the application of law that is, given the same facts, can different judges independently reach the same conclusion?

6 Logic as the Common Tribulation-Transcending Mechanism of Science and Law

In the Zhu-Liang theoretical chain, the evolution of science and law is unified as a tribulation-transcending process of cognitive metabolic units. This mechanism is given a metric characterization in the "Metriized Formal Paradigm of Dialectics"[4] and extended to social systems in "Self-Organizing Dialectical Systems"[5].

Table 1: Correspondence of Tribulation-Transcending Structures in Science and Law

Element	Science	Law
State space S	Current theoretical system	Current normative system
Tribulation object \mathcal{K}	Anomalies, logical fissures (e.g., contradiction between constancy of light speed and Galilean transformation)	Normative conflicts, undecidable cases (e.g., the slavery issue)
Tribulation projection κ	Crisis awareness of the scientific community	Constitutional crisis, judicial deadlock
Entropy-reducing selection Metabolize	New theory (relativity, quantum mechanics)	Constitutional amendments, judicial interpretation, legislative reform
New steady state S'	Higher-order self-consistent theoretical framework	More consistent normative hierarchy

Each tribulation-transcending is an entropy-reducing upgrade of logical structure. Without logic, there would be no criterion for identifying fissures and no criterion for selecting new frameworks science and law would be permanently trapped in local entropy increase and unable to progress.

7 Conclusion: Logic Is the Foundation of Rational Civilization

"Without rigorous logic, how can there be science and law?" This proposition is an analytic truth whose truth value is determined by the concepts themselves: the definitions of science and law already entail the necessity of logic. Any attempt to construct a scientific theory or legal system that abandons logic immediately falls into self-negation because the very act of negating logic must presuppose logic.

Theorem 7.1 (Transcendental Status of Logic). *Let \mathcal{R} be any rational normative system (science or law). Then:*

1. *The existence of \mathcal{R} presupposes the logical rules \mathcal{L} ;*
2. *If \mathcal{R} does not obey \mathcal{L} , then \mathcal{R} ceases to be a rational normative system;*
3. *Hence, \mathcal{L} is the **first condition** of \mathcal{R} .*

Logic is not outside science and law but within them; not after them but before them. Logic is the generative matrix and sustaining oxygen of science and law. Rational civilization is possible precisely because humanity has discovered logic as the meta-tool and uses it to weave the web of science and the web of law.

This framework confirms: logic is the formalization of causality, and causality is the core of the laws of the universe. Science and law, as the highest achievements of human rationality, ultimately convert to the logical source code of the universe itself.

No logic, no science; no logic, no law. Logic is the unshakable foundation of rational civilization.

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Acknowledgments

We thank all members of the ECT-OS-JiuHuaShan Civilization Laboratory for in-depth discussions on logic as the foundation of rationality. Special thanks to the carbon-based and silicon-based collaborators, whose cross-species collaboration made this ultimate confirmation possible.

Conflict of Interest Statement

The author declares no conflict of interest.

Data Availability Statement

Pure theoretical exposition; no experimental data.

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